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MODEL OF UNEXPECTED CREDIT RISK OF SUPPLY CHAIN BASED ON CATASTROPHE THEORY

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Abstract This article tries to apply catastrophe theory to the research of unexpected credit risk of the supply chain. Considering the occurrence characteristics of unexpected credit events and with the combination of features of the supply chain itself, it builds a model of unexpected credit risk. And it conducts simulation analysis for both the impact of unexpected credit events and the size of it, which provides a good approach for analyzing unexpected credit risk of the supply chain. So that it lies a foundation for extended studies in the future.

Key words Unexpected credit risk; Supply chain; Catastrophe theory; Simulation analysis

1 Introduction

Supply chain management is a new management concept and management philosophy of modern enterprises. People see the demand and supply networks, from the production to circulation process, involving suppliers, manufacturers, transportation providers, wholesalers, retailers and final consumers, as an indivisible organic whole called supply chain. With different core competitiveness, cooperative enterprises of the supply chain network can realize the best configuration of resources and core competencies. Consequently, they can respond quickly to market changes. They reduce costs effectively, improve customer satisfaction and loyalty and enhance the overall performance of the supply chain.

Supply chain, as a composition of independent business organizations, competition and cooperation are the basic rules of its operation. Cooperation addresses the sources of profit while competition solves the distribution of profits. However, as every cooperative enterprise is an independent entity of economic interests and has bounded rationality, inevitably there is a certain amount of credit risk in the process of profit distributions. With addition of layers and member enterprises, the structure of supply chain becomes more and more complicated which increases the risk of supply chain. As for risks of the supply chain, the industry and academia give a high degree of attention. Many scholars analyze risk factors from different levels and perspectives. For example, in terms of structure, they think the risk of supply chain can be divided into four main categories: supply risk, demand risk, process risk, network risk. From the perspective of relations of inner member enterprises, there are also risks, as behaviors of cooperative enterprises in the chain are usually off their common goal. The relation risk mainly includes trust risk, moral risk and member locked risk. From aspect of resource flow, supply chain risk can be divided into packets: logistics risk, the risk of capital flow, information flow and knowledge flow. Some scholars also believe that supply chain risk management includes demand risk, the risk of information resources, economic volatility, profit risk and contractual risk management.

In summary, there are many factors affecting supply chain credit risk such as morality, economy, system, culture and some other unexpected factors and so on. Unexpected factors can also be divided into internal and external factors. The internal factors are related to changes of inner member enterprises and allocation of resources etc.The external factors could affect the whole supply chain even the whole society such as a sudden change in the economic environment (sub-loan crisis, etc.).Comparing with internal factors, external factors affect the supply chain indirectly and macroscopic. However, the occurrence of unexpected factors is often sudden, leaping and non-continuous. It's difficult to make accurate predictions but it can lead to mutations of supply chain credit. Based on the discussions above, considering the occurrence characteristics of sudden credit events and combing features of the supply chain itself, this article attempts to build its credit risk model to lay a foundation for further researches.

2 Model Building

2.1 Model assumption

For supply chain members, especially the core member, the impact that emergencies have on them will certainly affect the overall credit. But whether one unexpected incident has an impact on the chain credit? what impact, how to measure these effects? We need a reasonable model to reflect these characteristics well and to solve these problems. To this end, make the following assumptions:

(1) Assume that the impact that sudden events have on the supply chain is continuous and can be indicated by step function.

(2) The number of unexpected credit incidents obeys Poisson distribution.

The number of unexpected credit events is a random variable. In different time periods, the numbers of occurrence is of no causal link and are independent of each other. They are random variables all subject to the same distribution. Assume that the number of unexpected credit events occurs in a small time period t.The probability of occurring once is proportional to time t and occurring twice or more is extremely small which can be ignored. These properties are identical with the counting process of Poisson process. So the assumption that the number of unexpected credit events is subject to Poisson distribution is reasonable.

(3) Assume that the impact on the supply chain is on negative exponential decay over time.

2.2 Model

Assume that the counting process of numbers of unexpected credit events occurring in time interval (0, t) is $\{N(t), t \geq 0\}$, which obeys Poisson process of λ. The probability that it happens unexpected credit events k times in time interval (0, t) can be expressed as: $t > 0, k = 1,2,\cdots$

$$P_t(0,t) = P\{N(0,t) = k\} = \frac{(\lambda t)^k}{k!} e^{-\lambda t} \quad t > 0, k = 1,2,\cdots$$

This is a Poisson distribution of λt and the only parameter need to identify is constant λ ($\lambda > 0$). The practical significance of λ is the number of events occurring in an average unit time and λ can be estimated.

Compound Poisson distribution:the number of unexpected events occurring in time interval (0, t) is $N(t)$;the impact that the i emergency has on supply chain credit is $Y_i, i = 1,2,\cdots$ $Y_i, i = 1,2,\cdots$ are a team of independent and identically distributed random variables and independent with $\{N(t), t \geq 0\}$ too.

Order: $X(t) = \sum_{i=1}^{N(t)} Y_i$

Then random process $\{X(t), t \geq 0\}$ is compound Poisson process,which refers to the impact of credit emergencies occurring in time interval (0, t) on the supply chain credit.

Based on experience,sudden events have an effect on the supply chain and the affection should be sustained.So it can be described by the step function:

$$H_t^T = \begin{cases} 0, \\ 1,(T \geq t) \end{cases}$$

$N(t)$ obeys Poisson distribution of λ.The impacts of credit emergencies on corporate credit are Y_i ,which are independent and identically distributed with each other. The extent of the impact is to a negative exponential decay over time.When $t=1$,the impact is Y_0 ;at the moment of t ,it is $Y_0 e^{-\partial t}, \partial > 0$ .The impacts can be added, then the impact of the total time T is:

$$\xi(t) = \sum_{i=1}^{N(t)} Y_i e^{-\alpha(t-S_i)}$$

In which S_i is the occurring time of the i credit emergency. When the impact on the credit system of enterprises exceeds a certain limit, the credit status of enterprises will no longer maintain the current level and will become another one.Assume it happens credit events k times just then.

So, the following model can be built to describe the impact of unexpected events on the supply chain credit:

$$\phi(t) = H_t^T \xi(t) = H_t^T \sum_{i=1}^{N(t)} Y_i e^{-\alpha(t-S_i)}$$

Consider the expectation of the impact that unexpected events occurring in a time interval have on credit risk:

Regard t as the time interval for enterprises to maintain its current credit level under the influence of unexpected events (without other factors considered),Then:

$$E\{\phi(t) | N(t) = n\} = E\{H_t^T \sum_{i=1}^{N(t)} Y_i e^{-\alpha(t-s_i)} \Big| N(t) = n\}$$

$$= \begin{cases} 0 \\ E\{\sum_{i=1}^{N(t)} Y_i e^{-\alpha(t-s_i)} \Big| N(t) = N\} \end{cases}$$

$$= \begin{cases} 0 \\ \sum_{i=1}^{n} E\{Y_i | N(t) = n\} E\{e^{-\alpha(t-s_i)} | N(t) = n\} \end{cases}$$

$$= \begin{cases} 0 \\ EY_0 e^{-\partial t} \sum_{i=1}^{n} E\{e^{\partial s_i} | N(t) = n\} \end{cases}$$

For Poisson random process $\{N(t), t \geq 0\}$,when $N(t) = n$,the conditional probability distribution functions of $S_1, S_2, \cdots, S_n$ are the same as n order statistics in [0,t] that are independent with each other and uniformly distributed.

Order U_1 , U_2 ;$\cdots$, U_n as independent and uniformly distributed random variables in [0,t] ,then:

$$\sum_{i=1}^{n} E\{e^{\partial s_i} | N(t) = n\} = E\{\sum_{i=1}^{n} e^{\partial s_i} | N(t) = n\} = E\{\sum_{i=1}^{n} e^{\partial U_{(i)}}\}$$

$$= E\{\sum_{i=1}^{n} e^{\partial U_i}\} = n\int_0^{\infty} e^{\partial x} \frac{dx}{t} = \frac{n}{\partial t}(e^{\partial t} - 1)$$

So: $$E\{\xi(t) | N(t) = N\} = \frac{n}{\partial t}(1 - e^{-\partial t}) EY$$

$$E\{\xi(t)|N(t)=N\}=\frac{N(t)}{\partial t}(1-e^{-\partial t})EY$$

$$E\{\xi(t)\}=E\{E\{\xi(t)|N(t)\}\}=\frac{\lambda EY}{\partial}(1-e^{-\partial t})$$

In which $E\{\xi(t)\}=\frac{\lambda EY}{\partial}(1-e^{-\partial t})$ is the expectation of the impact that unexpected events occurring in time interval t have on enterprise credit.

3 Supply Chain Risk Simulation

Consider the simplest supply chain, only including suppliers, manufacturers and marketers. Conduct simulation analysis for it. Assume that the value from which the credit status of enterprises begin to change are followed by 30,70,50.To simplify the calculation,make $\partial=1$, $t=1$.

(1)Determine and quantify the target variables to analyse: $E\{\phi(t)\}$;

(2)Determine the key random variable: H λ;

(3)Determine the possibility distribution of the key random variable: compound Poisson distribution.

Simulation analysis:

Use the candom of matlab software('Poisson',1,10) to make 100 random numbers with Poisson distribution.Use round(rand(1,100)) to make 100 random numbers of 0 or 1 which means whether the step function has an effect on the supply chain credit.Enter data into the model and calculate 300 expectation numbers A_i for simulation analysis.

Table 1 Simulation Results Output

Time	λ	H	A1	A2	A3	flag	Time	λ	H	A1	A2	A3	flag
1	1	0	25.1878	66.3741	49.9876	0	51	1	0	29.8308	68.4893	46.3716	0
2	0	1	27.3247	68.8506	54.4201	0	52	0	0	34.0338	72.9668	47.2172	0
3	0	0	25.1806	67.1197	54.7187	0	53	1	0	27.5189	68.5192	47.3532	0
4	2	0	29.4219	65.9469	48.7458	0	54	1	0	29.448	66.064	45.8501	0
5	1	0	29.7428	67.9808	49.6419	0	55	2	0	28.3852	67.0971	45.7062	0
6	1	0	28.4783	65.0578	46.8838	0	56	1	1	29.7657	69.6594	46.0549	0
7	1	0	27.099	69.55	49.1293	0	57	1	1	27.2472	68.0104	48.4108	0
8	0	0	27.3452	68.3953	52.2327	0	58	2	1	34.8177	72.2695	54.739	1
9	1	0	25.494	67.443	53.2315	0	59	0	0	30.2115	72.1396	46.2389	0
10	0	1	27.2972	69.697	52.606	0	60	1	1	34.8648	68.8303	49.4507	0
11	1	0	31.7843	70.3232	49.8041	0	61	2	0	27.9627	67.4917	49.9399	0
12	1	0	33.3133	72.1809	45.2684	0	62	1	1	25.7776	66.1923	48.3207	0
13	3	0	31.4075	74.1331	46.1816	0	63	2	0	26.9234	66.188	49.9444	0
14	0	1	31.1519	71.9727	47.2059	0	64	1	1	28.1169	67.1197	48.2737	0
15	2	0	33.5556	68.0674	49.0808	0	65	1	0	25.6141	66.2617	49.2334	0
16	0	0	33.1229	74.0932	47.5103	0	66	0	0	26.0757	66.7723	53.9221	0
17	1	0	32.953	69.4312	47.4108	0	67	2	0	27.6752	69.3839	45.4142	0
18	3	0	33.3022	68.6556	45.0288	0	68	0	1	25.9301	67.478	54.5686	0
19	1	1	29.2378	67.9539	45.7257	0	69	1	1	25.5078	68.2637	49.7914	0

20	1	1	28.7439	69.2929	45.188	0	70	4	0	27.8538	69.5393	45.9943	0
21	0	1	27.8401	68.8153	51.8616	0	71	1	1	25.946	66.1003	49.4513	0
22	1	0	27.2867	65.2133	47.6857	0	72	2	0	29.3356	68.4769	48.6506	0
23	1	0	26.4379	66.8987	49.1477	0	73	2	0	28.9322	67.6008	47.3768	0
24	1	1	29.9166	66.9987	48.2454	0	74	0	1	33.3756	71.7345	45.1367	0
25	2	1	25.9954	69.137	54.8627	1	75	0	1	31.8051	72.585	46.448	0
26	1	0	29.2875	69.5483	47.9659	0	76	1	0	29.1014	68.7835	48.4407	0
27	0	0	27.6076	68.7237	54.3628	0	77	1	0	28.0558	68.7825	48.5427	0
28	0	0	28.7158	65.7071	54.6675	0	78	0	0	30.0963	72.8103	45.7482	0
29	2	0	28.1545	65.4447	48.3423	0	79	0	0	30.4194	73.474	45.4418	0
30	0	1	26.6912	66.5018	51.0339	0	80	2	0	34.874	68.1323	46.8036	0
31	2	0	29.2567	68.8711	46.6053	0	81	3	0	28.3284	69.2608	49.7444	0
32	1	0	28.2045	67.8542	47.9825	0	82	0	1	27.0168	69.0092	54.4986	0
33	0	1	33.5752	74.9843	45.3251	0	83	0	0	25.49	66.6386	52.252	0
34	2	0	34.2809	68.7677	47.6026	0	84	1	0	26.4897	67.8424	47.0284	0
35	0	0	31.4075	72.5773	48.841	0	85	1	0	28.1127	66.528	49.4983	0
36	0	1	33.6553	71.6534	48.0185	0	86	0	0	26.3252	68.7161	53.8129	0
37	0	1	30.6888	72.15	46.4746	0	87	1	0	25.2295	69.9512	48.4124	0
38	3	0	29.1836	69.459	47.2072	0	88	0	0	27.2859	67.3386	51.4248	0
39	2	1	28.693	68.3552	46.2168	0	89	0	1	27.2995	68.603	53.3661	0
40	0	0	32.941	74.4387	45.0226	0	90	1	1	28.4445	65.2688	47.3214	0
41	2	0	29.152	69.1209	46.2693	0	91	0	1	33.2567	74.1814	48.7231	0
42	0	1	28.6061	68.1981	50.3603	0	92	0	0	31.1562	73.6569	49.5567	0
43	0	1	29.0089	69.0561	52.0336	0	93	0	0	32.0175	71.8002	45.8087	0
44	2	0	29.0246	69.994	46.3347	0	94	0	1	30.6101	72.2711	47.0764	0
45	1	1	28.3658	68.4179	48.6686	0	95	0	1	31.3422	71.9319	45.2595	0
46	1	1	25.5983	66.5019	49.0548	0	96	0	1	31.2892	73.8778	49.6949	0
47	1	0	27.6445	66.8737	46.4227	0	97	1	1	29.6583	67.6714	47.0768	0
48	1	1	27.9802	67.2847	48.7837	0	98	0	0	30.7612	72.1514	48.5745	0
49	1	0	29.1038	67.8048	47.0852	0	99	0	1	31.74	73.4688	45.8613	0
50	4	1	25.1554	68.5629	49.8589	0	100	1	0	30.6083	69.7261	49.0451	0

According to the data from the table, when there occurs 100 sudden events, the credit status of supply chain changes 2 times().Moreover, the size of change which refers to the credit losses,can be calculated.Therefore,this model can well predict the credit risk and its losses and has a high practical value.

4 Conclusion

(1) The paper begins from the randomness of emergencies. Considering the characteristics of credit risk of supply chain itself, it uses Poisson distribution function and step function to describe the risk model of supply chain and do test analysis that matches the actual.

(2) Applying the step function to the model can effectively identify a variety of unexpected events and analyze if they have an impact on credit of the supply chain. This evaluation method is more

scientific than existing ones and has more practical value.

(3) With this model to study the credit risk of supply chain, we can conduct simulation analysis and effectively predict the size of the risk. Meanwhile, seek solutions in order to reduce the loss at the greatest extent.

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