

# Study on Management and Compensation Mechanism of Transboundary Water Pollution Based on Game Theory

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**Abstract:** River pollution in China is strikingly represented by transboundary water pollution. The article analyzes the issue of transboundary water pollution based on game theory angle, constructs a non-cooperative model of target quota reduction method and proposes league game model allowing transferring pollutant and reduction of cooperative management. In order to guarantee the stability of league game, Nash method is adopted as fair profit allocation in cooperative regions and paid compensation obtained by different cooperative alliance is further determined. In this way, economic superiority in different areas can be developed and the overall gains of the valley can reach to the maximum. The conclusion is of great significance for avoiding the old way of river pollution control in China as well as pushing cycling economy development.

**Key words:** Transboundary water; Cooperative management model; Nash equilibrium; Compensation payment

## 1 Introduction

Water pollution preventive treatment, especially transboundary water pollution control, is a comprehensive systematic project, involving lots of profit principals<sup>[1]</sup>. In China, a great number of rivers cross two or more administrative regions. In different administrative regions of the same river, their profits are usually different. Sufficient clean water is essential condition for the development of downstream region, while upstream region possibly takes its geographical advantage to make full use of water resources but emits pollution into water. At present, different regions focus on pollution control of their own scope, which is difficult to realize effective result. For water pollution control, cooperation among different regions should be strengthened to cultivate regional superiority and establish effective incentive mechanism to promote regional cooperation so that harmonious development of economy, environment and society can be achieved.

Transboundary water pollution can be studied from macroscopic and microscopic levels. As macroscopic study mainly concentrates on management mechanism, it is not the key point of this paper.

Microscopic study focuses on the calculation and measurement of economic loss caused by water pollution as well as cooperative management mechanism. In recent years, with the increasing deterioration of transboundary water pollution in China, by economic calculation and from the scale of whole country and some regions, domestic scholars got the water pollution economic loss with sub-item sum method, established a series of mathematical model and made important progress in calculating economic loss of water pollution, such as Li Jinxiu and Xu Songlin theoretically proposed the calculating model of economic loss of general water pollution and the method of parameter determination<sup>[2]</sup>. Besides, taking economy-developed but seriously-polluted Taihu Valley as an example, they also carried out some practical application. However, lacking a set of standardized decomposition method of economic loss and non-unified setting of sub-items, comparability of these kinds of research results is less and reliability is eliminated to certain extent.

On the study of cooperative control and relative compensation mechanism, foreign scholars Jorgensen, Georges Zaccour<sup>[3][4]</sup>, etc, domestic scholars Lei Yutao<sup>[5]</sup>, Su Fengjuan and Guo Chengwei<sup>[6]</sup>, etc, all made some achievements and mainly concluded that regional cooperation is an effective way of treating environment pollution. As for compensation mechanism, many scholars both home and abroad regard it as the approach controlling pollution. The Principle is that the party causing pollution pays for it and the party gaining profit offers compensation. i.e. if ecological management and protection of upstream lays obvious positive environmental effect on downstream, downstream offers appropriate compensation to upstream; if upstream does not carry out effective pollution control, it needs to compensate downstream. In Laws and Regulations of Transboundary Water Pollution, Zeng Wenhui decided maximum and minimum of compensation by mapping<sup>[7]</sup>. Two parties negotiate the compensation amount and decide the final proposal.

Currently, most studies on cooperation management and compensation mechanism of valley

pollution are separated and joint study on both is quite few. The representative overseas is Jorgen and those who are at home are Zhao Laijun and Li Huaizu. Jorgensen and Gzacco established differential game model of pollution control between two countries and designed dynamic transferring incentive mechanism promoting long-run cooperation of two countries<sup>[8]</sup>. This model took pollution generating amount of various regions as unknown condition and through game model analyzed the optimal pollution amount of those regions under the cooperative and non-cooperative mechanism. Zhao Laijun and Li Huaizu constructed an order strategic model of dispute on transboundary water pollution, which empirically analyzed and contrasted the efficiency of pollutants deducting instruction quota management mechanism as well as regional cooperative management mechanism on the basis of transboundary water pollution of Huaihe Valley, contributing a lot for future study.

## 2 Analysis of Coordination of Transboundary Water Pollution

### 2.1 Analysis of transboundary water pollution from the perspective of externality

From external angle, the negative externality caused by transboundary water pollution is due to socialization of private cost during the process of seeking the minimum of it by economic entity. Generally, pollutants are inevitable in production. There are two ways to do with the pollutants: (1) Managing and discharging them when they're harmless to the environment; (2) directly emitting them to the environment. Management of the pollutants needs large amount of labor power and material resources, increasing the expenditure as part of private cost.<sup>[9]</sup> Motivated by profit maximization, producers usually choose not to control pollutants. Thus they are directly emitted into environment, which saves the expenditure and reduces the private cost. However, pollutants entering water will cause environment pollution, doing harm to people downstream or causing certain social economic loss, i.e. extra social cost. In this way, social cost of treating pollution is "saved", while the society pays more extra cost, and brings about the socialization of private cost.

### 2.2 Analysis of transboundary water pollution from the perspective of Game Theory

Since valleys cross lots of different provinces, cities and government organizations, there must be lots of pollutant dischargers. Suppose one valley crosses n regions, factually these n people are gaming. They will choose their maximum strategies according to the profit maximum to discharge pollutants or control them. If no one discharges pollutants, environment will not be polluted and the society will obtain the maximum profits. However, this is incredible, because if one does not discharge pollutant, control of it needs cost; if one shares the same environment with others, one's effectiveness becomes low. Therefore, all people discharge pollutants. This is the dilemma of n persons' game.

## 3 Non-cooperative Model of Target Quota Reduction

The hypothetical condition for the model establishment is that all the valley-crossing governments have already realized that economy development with environment sacrificing is not beneficial to the regional sustainable development and all governments prohibits to transfer pollutants to downstream areas. Under this precondition, maximum profits are pursued.

The purpose of this model is to control water pollution, improve valley water environment, completely and systematically investigate and analyze social economical development level and present situation of water environment of valleys as well as divide water functional zones. In addition, it aims to calculate pollution-holding ability of water functional zones depending on the thought of volume control, systematic plan and Nash equilibrium of non-cooperative game and finally establish the distribution model of pursuing regional maximum profits.

### 3.1 Model establishment

Suppose river crosses region  $i=1, 2, \dots, n$ ,  $e_i$  is the pollutant amount produced in region  $i$ ;  $U_i(e_i)$  represents profit function of pollutant due to  $e_i$  produced in region  $i$ ;  $p_i$  represents reduced pollutant amount in region  $i$ ;  $C_i(p_i)$  represents cost function consumed in reducing pollutant;  $s_i$  represents pollutant deposition in region  $i$ , related with the amount of pollutant discharging treated in different regions as well as water self-purification coefficient  $\alpha$ , ( $0 < \alpha_i < 1$ ), and  $s_i = (1 - \alpha_i)(e_i - p_i)$ ;  $D_i(s_i)$  represents loss cost function caused by deposition pollutant in region  $i$ ;  $w_i$  represents maximum environment volume of certain pollutant required by water functional zones in region  $i$ .

The target function is:

$$\max Z_i = U_i(e_i) - C_i(p_i) - D_i(s_i)$$

The constraint condition is:

$$\begin{cases} (1 - \alpha_i)(e_i - p_i) \leq w_i \\ e_i - p_i \geq 0 \\ e_i \geq 0 \\ p_i \geq 0 \end{cases}$$

The problem will be solved by non-linear constraint optimization method and the optimal policy  $e_i$ ,  $p_i$ , and target function  $Z_i$  will be obtained. In the solution,  $e_i$  is the optimal amount of pollutant in this region,  $p_i$  is the optimal reducing amount of pollutant in this region,  $Z_i$  is the maximum social profit in this region.  $Z$  is the social profit sum in the regions crossed by the valley under the model, i.e. total social profit of the valley.

$$Z = \sum_{i=1}^n Z_i$$

### 3.2 Model analysis and the conclusion

From the conclusion of the model above, establishing distribution mechanism, to certain degree, holds back free discharging behavior of regions and reduces disputes of water pollution. Whereas the method of independently reducing quotas and forbidding transferring reduction of pollutant does not consider the discrepancy of the economic structure in different regions, the profit function determined by economic development level in developed regions and developing regions, cost function of pollutant reduction and different loss cost function, resulting in unfulfilled cultivation of the social effective resources. Therefore, the obtained total social profit of the valley from this model is not the optimal solution of Pareto.

## 4 League Game Model of Joint Management of Pollutant Transferring Reduction

Suppose there is an administrative organization in the studies valley with its function to fully develop the discrepancy of all the regions, realize the maximum profit of the whole valley and not to damage the water environment function of the valley and different regions. The league game principle established by the above joint administrative organization and regional government above is that downstream regional governments have the obligation to manage the pollutant transferred from upstream governments and also the obligation to use the environment capacity offered by the upstream governments.

### 4.1 Model establishment

Model variable is the same with the above part and new variable  $M_{ij}$ , ( $i < j \leq n$ ) is introduced, representing the pollutant amount transferred by region  $i$  to downstream regions or environment capacity.  $M_{ij} \geq 0$  represents pollutant amount it transferred to downstream,  $M_{ij} \leq 0$  represents environment capacity offered by regional government  $i$  in improving pollution management; in  $M_{ki}$ , ( $1 \leq i < k$ ),  $M_{ki} \geq 0$  represents pollutant amount in regional  $i$  transferred by upstream governments, otherwise represents the environment capacity offered by upstream governments.

Pollutant deposition in region  $i$ :

$$s_i = (1 - \alpha_i) \left( e_i - \sum_{j=i+1}^n M_{ij} + \sum_{k=1}^{i-1} M_{ki} - p_i \right)$$

$w_{\max}$  is the total environment capacity of the valley:

$$w_{\max} = \sum_{i=1}^n w_i$$

General target function of the studied valley:

$$\max Z^* = \sum_{i=1}^n [U_i(e_i) - C_i(p_i) - D_i(s_i)]$$

The constraint condition is:

$$\begin{cases} g_i = (1 - \alpha_i) \left( e_i - p_i - \sum_{j=i+1}^n M_{ij} + \sum_{k=1}^{i-1} M_{ki} \right) \leq w_i, i = 1, 2, \dots, n \\ e_i \geq 0, i = 1, 2, \dots, n \\ p_i \geq 0, i = 1, 2, \dots, n \end{cases}$$

Some results will be found, i.e. optimal pollutant production  $e_i^*$  of region  $i$ , optimal pollutant

reduction amount  $p_i^*$ , pollutant amount received from the transference of upstream  $(M_{1i}, M_{2i}, \dots, M_{ki})$ , pollutant amount transferred to downstream  $(M_{i,i+1}, M_{i,i+2}, \dots, M_{in})$ . Meanwhile, maximum social profit  $Z^*$  will be obtained.

According to

$$Z_i^* = U_i(e_i^*) - C_i(p_i^*) - D_i(s_i^*)$$

regional profit  $Z_i^*$  under joint league will be found,

$$Z^* = \sum_{i=1}^n Z_i^*$$

#### 4.2 Model analysis and its conclusion

In this joint model, studying from maximum profit of the whole valley fully considers the discrepancy of economy development of various regions, as well as the complementary advantage of profit function, cost function of pollutant reduction and loss cost function. It also gives consideration to the sustainable development of economy and environment, which realizes the Pareto optimization of the valley's whole profit.

From the practical sense, environment cooperation is prior to non-cooperation, i.e. cooperation surplus should be produced and obviously  $Z^* > Z$ .

### 5 Cooperative Profit Allocation and Compensation Payment

#### 5.1 Model establishment

##### 5.1.1 Cooperative profit allocation model

All figures should be numbered with Arabic numerals (1,2,...n). All photographs, schemas, graphs and diagrams are to be referred to as figures. Line drawings should be good quality scans or true electronic output. Low-quality scans are not acceptable. Figures must be embedded into the text and not supplied separately. Lettering and symbols should be clearly defined either in the caption or in a legend provided as part of the figure. Figures should be placed at the top or bottom of a page wherever possible, as close as possible to the first reference to them in the paper.

From the practical sense, environment cooperation is prior to non-cooperation and obviously  $Z^* > Z$ . However, it cannot be guaranteed for  $\forall i, Z^* > Z_i$ .  $Z^*$  is the profit in environment cooperation regions,  $Z_i$  is the non-cooperative profit. When  $Z^* > Z_i$  exists, person i in the game refuses cooperation. To promote environment cooperation, two problems concerned by all the parties must be solved: (1) feasibility of cooperation, i.e. guaranteeing every entity can benefit from the cooperation; (2) fairness of cooperation, i.e. whether every entity will be satisfied with the final effects. If cooperation is adopted, the cooperation should be stable. Thus the feasibility and fairness of the cooperation must be guaranteed, i.e. every entity can benefit from the cooperation, which needs to re-allocate the total social profit under cooperation.

This paper selects "Nash method" as the allocation method of cooperative fair profit. This method, through comparing the profits brought by effective distance of situation points under cooperation and non-cooperation, is represented as the following:

$$\max Y = \max \prod_{i=1}^n [Z_i^{**} - Z_i]$$

The constraint condition:

$$\begin{cases} \sum_{j=1}^n Z_j^{**} = Z^* = \sum_{i=1}^n Z_i^* \\ Z_i^{**} \geq Z_i, i = 1, 2, \dots, n \end{cases}$$

$Z_i^{**}$  is the re-collocative profits of region i under cooperation league,  $Z_i$  is the profit of appointed quota reduction under non-cooperative model,  $Z^*$  is the total profit of cooperation league.

##### 5.1.2 Compensation payment model

Suppose  $SP_i$  is the compensation payment of region i. After payment, the final profit of region i is

$$\begin{aligned} Z_i^* + SP_i &= Z_i^{**} \\ \sum_{i=1}^n SP_i &= \sum_{i=1}^n (Z_i^{**} - Z_i^*) = 0 \end{aligned}$$

Suppose  $f_{ki} (1 \leq k \leq i)$  is received compensation amount of region i due to transferred pollutant from upstream regions;

$f_{ij} (1 \leq j \leq n)$  represents compensation amount paid by region  $i$  due to transferring pollutant to downstream regions; so

$$SP_i = \sum_{k=1}^{i-1} f_{ki} - \sum_{j=i+1}^n f_{ij}$$

Thus, linear equations are constructed:

$$\left\{ \begin{array}{l} -f_{12} - f_{13} - \dots - f_{1n} = SP_1 \\ f_{12} - f_{23} - \dots - f_{2n} = SP_2 \\ \dots\dots\dots \\ f_{1i} + f_{2i} + \dots + f_{i-1,i} - f_{i,i+1} - \dots - f_{in} = SP_n \\ \dots\dots\dots \\ f_{1n} + f_{2n} + \dots + f_{n-1,n} = SP_n \\ \sum_{i=1}^n SP_i = 0 \end{array} \right.$$

The single digit of the unknown number of the equations is  $\frac{n(n-1)}{2}$ , therefore the equations must have non-zero solution.

**5.2 Model analysis and its conclusion**

From the above model, compensation payment is just a kind of effectiveness transference of people in the game of cooperation league. In practice, Nash method can be adopted to re-allocate total profit of cooperation league and also can transfer the effectiveness in different regions to guarantee that the final profit of each region is determined by Nash. This cooperation profit allocation and compensation payment method satisfies individual rationality and collective rationality. Thus cooperation can be guaranteed to go smoothly and stability of cooperation league can also be guaranteed.

**6 Conclusion**

On the basis of analyzing regional social economical variable of transboundary water pollution, This paper constructs non-cooperative model of target quota reduction as well as cooperative management model of league game starting from the practical demands of transboundary water pollution control in China. In order to guarantee the stability of league game, Nash method is adopted, as fair profit allocation method of each region under cooperation, further deciding the cooperation league profit and paid compensation of each regions and providing important methods and thinking for trans-regional cooperation of valleys as well as the win-win of the whole valley.

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