System Model and Analysis of Three-dimensional Dynamics for Flexible Multi-body System

Shohei Yamamoto[1], Shigeyuki Haruyama[2], Tadashi Sumimoto[2], Junji Kaneko[2]
1 Department of Science and Engineering, Yamaguchi University, 2-16-1 Tokiwadai, Ube-shi, Yamaguchi, Japan
2 Management of Technology, Yamaguchi University, 2-16-1 Tokiwadai, Ube-shi, Yamaguchi, Japan
(E-mail: r048ve@yamaguchi-u.ac.jp, haruyama@yamaguchi-u.ac.jp, sumitad0714@yahoo.co.jp, kaneko@yamaguchi-u.ac.jp)

Abstract: In product design, improvements in a design are based on the design information decided upon in a later stage of the design process; hence, the success of a product and the time required to develop the product are influenced by the quality of design at the abovementioned stage.In previous studies, I suggested the concept of a system model that combines simplicity and generality and verified the effectiveness of this model because I intended to rebuild the function design for focusing the concept on an object, and not on a regressing process, development being possible to spiral up steadily. In this paper, I examine the effectiveness of the abovementioned system model using a specific case because I intend to suggest a methodology for expanding the application of the abovementioned system model to the dynamics analysis of a three-dimensional structure, such as a robot or a crane.
Key words: System model; Flexible multi-body system; Analysis of three-dimensional dynamics

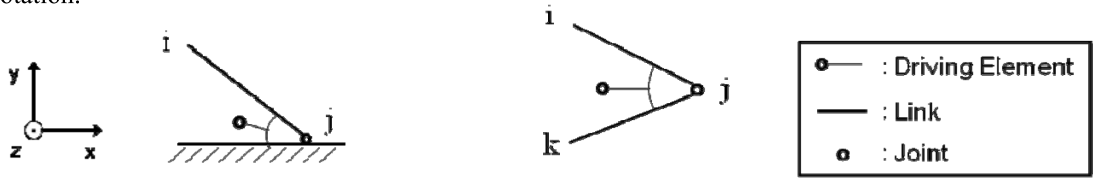
1 Introduction

Conceptual design in a later stage of product design is characterized by many ambiguities in the design information despite the fact that there is a high degree of design freedom at this stage. However, in product design, improvements in design are based on design information decided in a later stage of the design process; hence, the success of the product and the time required to develop the product are influenced by the quality of design at the abovementioned stage. In previous studies, I suggested the concept of a system model that combines simplicity and generality and verified the effectiveness of this model because I intended to rebuild the function design for focusing the concept on an object considered in the previous methods of design engineering and design, and not on a regressing process, development being possible to spiral up steadily. Further, I examined the effectiveness of the system model with respect to inverse engineering on the electric tools available in the market.[1][2] In this paper, to examine the effectiveness of the abovementioned system model, I apply this system model to the dynamics analysis of a three-dimensional structure. Traditionally, a multi-body system model that contains a rigid link has been used in the dynamics analysis of a three-dimensional structure, such as a robot, a crane, or space equipment. In fact, such a structure tends to be flexible; hence, it is difficult to accurately estimate the behavior of a control system built by a multi-body model of a rigid link structure. Further, it is practical to carry out a linkage analysis of flexible multi-body systems. In the linkage analysis, software such as RecurDyn and Adams is used. However, such software cannot be applied to certain specific areas for developing a unique system, such as research and development. Hence, I suggest the concept of a system model that combines simplicity and generality with general-purpose software such as MATLAB or SCILAB.

2 Basic Concept

Linkage analysis methods are used for the formulation of a beam structure having a large displacement and a large rotation, by a finite element method. Some of these analysis methods are the floating reference frame method and the rotational vector method. On the other hand, the linkage analysis of a flexible multi-body system and the finite element method used for a normal strength analysis are very different from each other in terms of the main function of the driving element, as shown in the figure below.

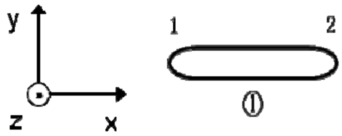
In the dynamics analysis of linkage including a drive and a control system, the angle or the movement between the elements is realized by a pin or a slide. Hence, the system is often controlled by the motor torque. In Figure 1 (a), the drive unit rotates the beam i-j around a fixed point j on the floor. In Figure 1 (b), the drive unit rotates the relative rotation angle of the beam i-j and the beam j-k. For the conventional modeling by the finite element method, an angular displacement variable a rotary drive contact with the ground is incorporated into the main analysis model. The angular displacement at node j of the rotary drives between the elements of the core components is incorporated into either element rotation.



(a) Rotary drive contact with the ground (b) Rotary drive between the elements

Figure 1 Rotary Drives

On the other hand, the angular displacement of the other element is typically calculated at the end because it is treated as a dependent variable between the relevant independent displacements. Therefore, it is necessary to incorporate a special element into the normal finite element model; the angular displacement of both the beam i-j and beam j-k in the rotary drives between the elements should be included. Therefore, simplicity and generality obtained by the finite element method disappear, and the analytical system development method becomes relatively complex.In this paper, for solving the equations of motion, I ensure that there is no problem in the numerical analysis; even the stiffness matrix is singular if the mass matrix is entered. Further, on the basis of the basic object-oriented concepts of the system model and the incorporation of the element stiffness matrix into the structure stiffness matrix, I suggest a join table of the nodes. First, for element ① with nodes 1 and 2 as shown below, the element stiffness matrix of element ①, whose node numbers are subscripted, is represented using small matrices and small vectors as follows. Here, the inside subscript of a small matrix represents the number of nodes, and the outside subscript represents the element number.



$$\begin{matrix} & 1 & 2 \\ 1 & [k_{11}]_{\text{①}} & [k_{12}]_{\text{①}} \\ 2 & [k_{21}]_{\text{①}} & [k_{22}]_{\text{①}} \end{matrix} \begin{Bmatrix} \{d_1\} \\ \{d_2\} \end{Bmatrix} = \begin{Bmatrix} \{f_1\}_{\text{①}} \\ \{f_2\}_{\text{①}} \end{Bmatrix} \qquad (1)$$

Figure 2 Element with Two Nodes

2.1 Traditional synthesis method

In the traditional synthesis method, a structure is resynthesized after the structure is factored and the element stiffness matrix is created considering a rigid coupling or pin connectors. I consider an elastic pendulum support frame in a plane problem to show the flow of the conventional synthetic methods.

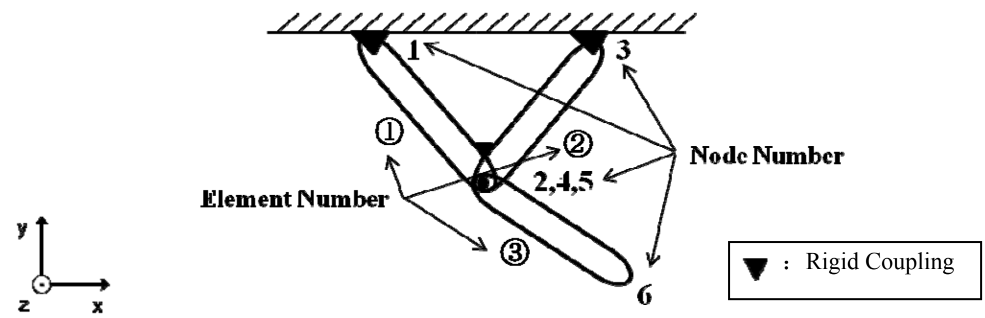


Figure 3 Elastic Pendulum Support Frame

The structure stiffness matrix prior to the restraint on the node as follows:

$$\mathbf{K} = \begin{bmatrix} [k_{11}]_{\textcircled{1}} & & [k_{13}]_{\textcircled{1}} & \\ & [k_{22}]_{\textcircled{2}} & [k_{23}]_{\textcircled{2}} & \\ [k_{31}]_{\textcircled{1}} & [k_{32}]_{\textcircled{2}} & [k_{33}]_{\textcircled{1}} + [k_{33}]_{\textcircled{2}} + [k_{33}]_{\textcircled{3}} & [k_{34}]_{\textcircled{3}} \\ & & [k_{43}]_{\textcircled{3}} & [k_{44}]_{\textcircled{3}} \end{bmatrix} \tag{2}$$

However, the element stiffness matrix of the pin connectors is used at node 3 of element ③. Moreover, the mass matrix is follows as in the same manner.

$$\mathbf{M} = \begin{bmatrix} [m_{11}]_{\textcircled{1}} & & [m_{13}]_{\textcircled{1}} & \\ & [m_{22}]_{\textcircled{2}} & [m_{23}]_{\textcircled{2}} & \\ [m_{31}]_{\textcircled{1}} & [m_{32}]_{\textcircled{2}} & [m_{33}]_{\textcircled{1}} + [m_{33}]_{\textcircled{2}} + [m_{33}]_{\textcircled{3}} & [m_{34}]_{\textcircled{3}} \\ & & [m_{43}]_{\textcircled{3}} & [m_{44}]_{\textcircled{3}} \end{bmatrix} \tag{3}$$

Then, the constraint process is given.

3 Synthesis with System Model

In the synthesis with the system model, each element is regarded as an object with a unique node number on the basis of the object-oriented tenet of the system model. Next, the structure is configured and placed together part by part. Therefore, the element stiffness matrix is created after assigning the same coordinates to the nodes placed in the same position. Finally, the join table of the nodes corresponding to the table of the nodal degrees of freedom and structure freedom is created by considering the fulcrum restraint and the pin connectors or slide coupling.

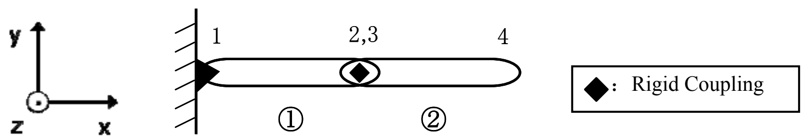


Figure 4 Cantilever

Table 1 Join Table of Nodes

		Degree of Freedom for Node		
		u	v	θ_z
Node Number	1	0	0	0
	2	1	2	3
	3	1	2	3
	4	4	5	6

u: x-displacement [m]
v: y-displacement [m]
θ_z: Rotation angle around z-axis [rad]

The bond stiffness is observed between node 2 and node 3, and the degrees of freedom of node 2 and node 3 are the same. Therefore, the serial number for each degree of freedom of node 2 and node 3 is the same.

3.1 Considering pin connector

With respect to the pin connector, because the rotational degrees of freedom increase between nodes, the join table of the nodes is as follows:

The coupling pin is between node 2 or node 4 and node 5, and the degree of freedom for the translational displacement of node 2 or node 4 and the degree of freedom of node 5 are the same.

3.2 Including drive element

In the previous linkage, the angle between the pin connecters is often controlled by the motor torque. Here, the driving element involves relative rotation and elements ② and ③, as well as the driving source angle.

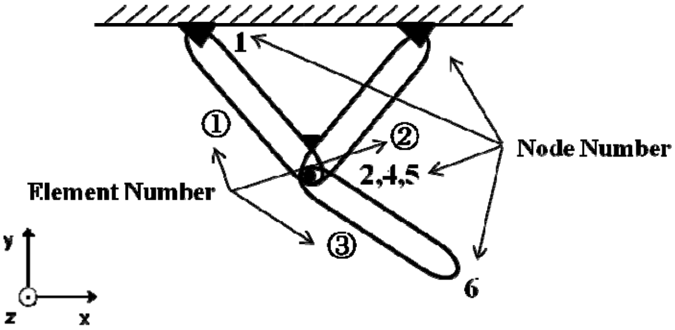
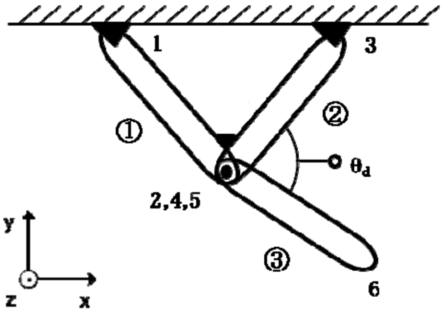


Figure 5 Elastic Pendulum Support Frame

Table 2 Join Table of Nodes

		Degree of Freedom for Node		
		u	v	θ_z
Node Number	1	0	0	0
	2	1	2	3
	3	0	0	0
	4	1	2	3
	5	1	2	4
	6	5	6	7

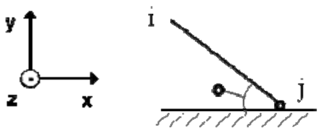


$$k_\theta \begin{bmatrix} R^2 & -R^2 & -R \\ -R^2 & R^2 & R \\ -R & R & 1 \end{bmatrix} \begin{Bmatrix} \theta_{z4} \\ \theta_{z5} \\ \theta_d \end{Bmatrix} = \begin{Bmatrix} M_{z4} \\ M_{z5} \\ M_d \end{Bmatrix} \quad (4)$$

Figure 6 Elastic Pendulum Support Frame

k_θ: Drive Shaft Stiffness [Nm/rad], R: Reduction Ratio, M: Torque [Nm]

A rotary drive contact with the ground is as follows:



$$k_\theta \begin{bmatrix} R^2 & -R \\ -R & 1 \end{bmatrix} \begin{Bmatrix} \theta_j \\ \theta_d \end{Bmatrix} = \begin{Bmatrix} M_j \\ M_d \end{Bmatrix} \quad (5)$$

Figure 7 Rotary Drive Contact with the Ground

Here, the join table of the nodes shown in Figure 6 is as follows:

Table 3 Join Table of Nodes

		Degree of Freedom for Node		
		u	v	θ_z
Node Number	1	0	0	0
	2	1	2	3
	3	0	0	0
	4	1	2	3
	5	1	2	4
	6	5	6	7
Drive Source Angle		0	0	8

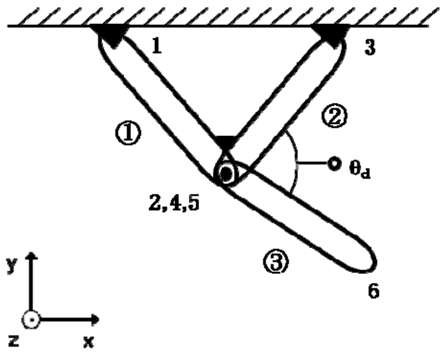


Figure 8 Elastic Pendulum Support Frame

4 Validating Effectiveness for Modeling

In this paper, a case study for verifying the effectiveness of the synthesis by the system model is discussed. I selected a ceiling crane for this case study. The ceiling crane, as shown in Figure 9, moves in the vehicle trajectory arranged along the walls on either side of the building. In general, it can be operated to generate three behaviors: winding, traversing, and traveling. Because it can ensure a wide range of work space, it has been used transporting heavy loads and parts in a machine shop.

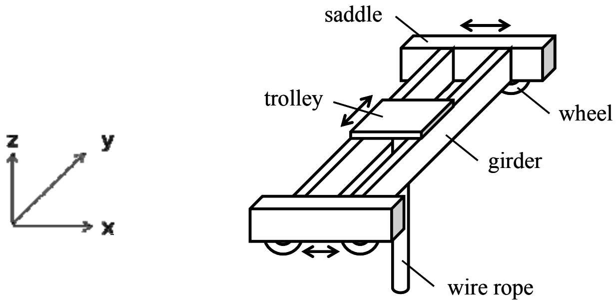


Figure 9   Schematic Representation of Ceiling Crane

The elements, node model, and join table of the nodes of this ceiling crane are as follows: In this paper, I model the ceiling crane for simulating the traveling behavior only. Further, I use a chain instead of a rope to verify the effectiveness of the proposed modeling. Here, the origin is the initial position of node 4.

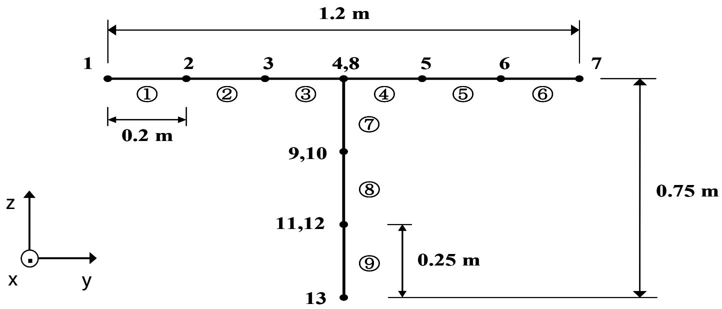


Figure 10   Ceiling Crane Model

Table 4   Join Table of Nodes

		Degree of Freedom for Node					
		u	v	w	θ_x	θ_y	θ_z
Node Number	1	1	0	0	2	3	4
	2	5	6	7	8	9	10
	3	11	12	13	14	15	16
	4	17	18	19	20	21	22
	5	23	24	25	26	27	28
	6	29	30	31	32	33	34
	7	35	0	0	36	37	38
	8	17	18	19	39	40	41
	9	42	43	44	45	46	47
	10	42	43	44	48	49	50
	11	51	52	53	54	55	56
	12	51	52	53	57	58	59
	13	60	61	62	63	64	65

u: x-Displacement [m]
v: y-Displacement [m]
w: z-Displacement [m]
θ_x: Rotation Angle around x-Axis [rad]
θ_y: Rotation Angle around y-Axis [rad]
θ_z: Rotation Angle around z-Axis [rad]

The member length of element ② to element ⑥, and element ⑦ and element ⑧ are the same as the member length of element ①, and element ⑨. The parameters of all the elements in the ceiling crane model and the simulated model are as follows: As a loading condition, a force of 500 N is exerted in the x-direction for node 1 and node 7. The gravitational acceleration is 9.81 m/s^2.

Table 5 Parameters of all elements in ceiling crane model

Mass m [kg]	1
Moment of Inertia I [kgm^2]	3.3×10^{-1}
Extensional Rigidity K [N]	$2.0\pi\times10^{5}$
Bending Stiffness for Each Axis I [Nm^2]	157
Torsional Stiffness for Each Axis G [Nm/rad]	157

Table 6 Parameters of Simulation

Analysis Time [ms]	50
Step Time [ms]	0.1

Simulation results for the ceiling crane model reveal that the parameters of the elements change as follows:

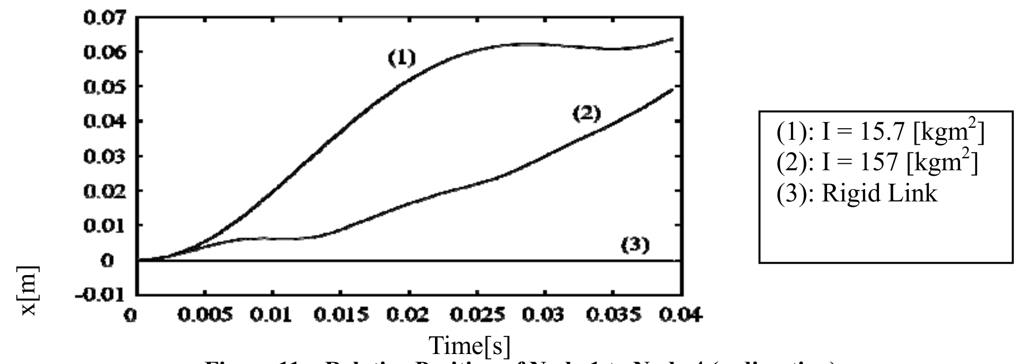


Figure 11 Relative Position of Node 1 to Node 4 (x-direction)

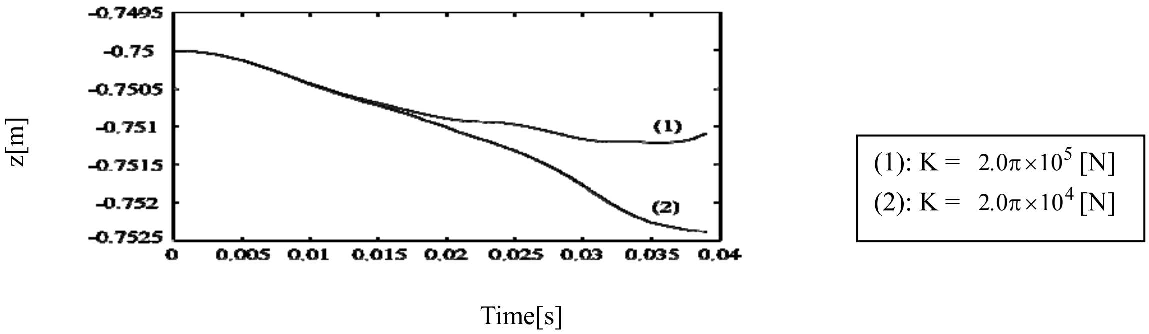


Figure 12 Displacement in the Z-direction for Node 13

The crane was not confirmed to oscillate when simulated as a rigid link structure, but it was confirmed to oscillate when it was simulated as a flexible link structure.

5 Conclusions

In this paper, I intended to expand the application of a previously developed system model; I suggested the concept of a system model that combines simplicity and generality. Further, I modeled a ceiling crane as a flexible structure of multi-body systems by the finite element method. I examined the theory of modeling by simulating and evaluating the behavior of the crane.

References

[1] Shohei Yamamoto, et.al. Reconstruction of Functional Design Method by System Model
[2] Michel Geradin, et.al. Flexible Multibody Dynamics. New York: Wiley-Blackwell, 2001