Knowledge Heterogeneity, Alliance Formation and the Evolution of Clusters

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Abstract In this paper we examine the evolution of network formation and innovation with special interest on the heterogeneity of firms in an industry. We present a model in which firms in an industry can innovate on their own or in alliance with each other. Alliance formation is based on the cognitive distance of firms: whether two firms form an alliance and their probability of success depends on their proximity in knowledge space. Knowledge on the other hand is modelled along two dimensions: breadth and depth. We use computer simulations to examine the dynamics of network formation and innovation in the model industry. The main result of our analysis is that in the present setting innovation falls in the long run and networks dissolve over time. In contrast, the heterogeneity of firms in terms of their knowledge bases does not decrease, but increases. This result contributes to our present understanding of network evolution with respect to heterogeneity and innovation.
Key words knowledge heterogeneity, innovation, network dynamics, alliance formation

1 Introduction

It is always impressive to go back to the roots of a given discipline when founding a study. In our case, however, it is not just an intention but has a coherent logical connection to the thoughts we are to present here. The root is nothing else than The Wealth of Nations by Adam Smith (Smith, 1959). In this work Smith emphasizes the essential role of the division af labour for the well-being of an economy. On the other hand, he argues that the division of labour is tightly connected to the accumulation of knowledge. In his view the division of labour is possible if people accumulate knowledge which makes them more efficient in a particular segment of production, thus knowledge forms the basis for knowledge accumulation. However, the connection is not one-directional, as a random division of labour, based on chance rather than knowledge leads to the accumulation of knowledge as people gain expertise in a field where they had no differentiated knowledge before. The context for this argument is that of the economic development and growth in wealth. The accumulation of knowledge eases the further division of labour through more efficient production and leads to economic growth by mutually advantageous exchanges.

After a long, but otherwise very fertile period of the neoclassical era in economics dynamic economic issues were integrated back to the main body of economic literature. The opening of this new vein was the neoclassical growth model of Robert Solow (Solow, 1956). The main contribution of the model is that if we include solely labour and physical capital into the set of production factors, economic growth can be only temporary: in the long run, the per capita production can not increase, as the growth rate of capital equals to that of labour force, thus capital intensity remains constant which leads to constant output per capita through a linearly homogenous production function. However, if (exogenous) technological change is integrated into the model (knowledge is included in the set of production factors), it can be shown that the long run growth rate of per capita output equals to that of the technological level. The main conclusion is quite similar to that of Smith: long run growth in the wealth of an economy comes from technological change (efficiency improvements), which is due to the accumulation of knowledge.

Although the Solow model proved very useful in understanding the processes of economic development, it seems somewhat lame as it leaves unexplained that part of growth, which is the most important: technological progress itself. This drawback of the Solow model led to the rise of endogenous growth theory, which tries to explain technological change inside the boundaries of its models rather than taking it as given. The main issue in endogenous growth literature is to somehow rule out the decreasing returns to production factors which impedes growth in the long run. Some models only draw on the cumulativeness of knowledge, i.e. knowledge is an input to the production of other goods, but the input of knowledge production is solely existing knowledge (Silverberg and Lehnert, 1994). One can fit among these models those where knowledge accumulation is the byproduct of some other economic activity, e.g. the learning-by-doing model of Arrow (Arrow, 1962). Although this cumulativeness of knowledge leads to sustainable long run growth, it does not account for the efforts made by economic agents in order to intentionally accumulate new knowledge. Models of this other kind, however, put the 'missing link' into the picture, emphasizing that resources must be allocated from other activities to knowledge production in order to gain new knowledge (Shell, 1967). These models are more convincing in contemporary economies where firms devote considerable resources to research and development, i.e. to knowledge accumulation.

All the models, mentioned before, build on a very strong assumption, namely that knowledge is not expropriable. This means that once a piece of new knowledge shows up at some point of the economy, it is freely and instantaneously accessible to all other agents. This could be a useful assumption, but requires revision. The instantaneous diffusion of knowledge is clearly counter-intuitive, as it requires time to be announced about novelties, gain access to the source of knowledge and to learn new things. Whether knowledge diffusion is costless, is an other issue. This view comes from the fact that knowledge is usually regarded as a public good, which is once generated (produced), is accessible to anyone with no possibility to exclude users (i.e. knowledge is non-excludable) and its use does not result in a decrease in the available stock (i.e. knowledge is non-rivalrous). While the second characteristic is clearly present, the first one (non-excludability) can be questioned. Once somebody creates new knowledge, she can retain it to herself by keeping it secret, or she can patent new knowledge thus excluding free riders from its use. The excludability of knowledge is tightly connected to its tacitness, to be discussed later. However, it is commonly agreed that excludability is present only temporarily: in the (very) long run, (roughly) all knowledge is accessible to anyone (i.e. secrets cease being secret, patents expire, etc.).

If knowledge does not diffuse without obstacles in the economy, it is worth examining how it diffuses. The literature on knowledge spillovers treats this issue quite thoroughly. Jaffe (1986) proves that innovation activity is not isolated in the economy as done by firms, but innovating firms use knowledge generated in other points in the economy as inputs to their knowledge generation processes. This clearly shows that knowledge is diffusing. Other studies, however, revealed that these knowledge spillovers are spatially bounded (Acs et al., 1992; Jaffe et al., 1993; Anselin et al., 1997). According to their findings, firms are more efficient in exploiting knowledge coming from other firms, universities and research institutes if they are located closer to these sources. Jaffe et al. (1993) shows that the localised effect of spillovers dies out through time, although this process is very slow. On the other hand, this finding is consistent with that mentioned above, i.e. knowledge can be considered a public good in the long run. These results from the knowledge spillover literature, however, refocus our attention to the issue of locality in economic growth.

Although proved empirically, the question remains why spillovers are spatially bounded. One of the main factors which are used to explain spatial concentration is the tacitness of knowledge. The distinction between tacit and codified knowledge comes from Polanyi (1966), although in the contemporaneous literature its meaning and use is somewhat blurred (De Carvalho et al., 2006). Codified knowledge is easily formalized, and thus easily communicated through high distances without loss of information or meaning. Tacit knowledge, however, can not be formalized, thus its transfer requires direct face-to-face interactions between the sender and the receiver, which in turn needs spatial proximity among agents. Thus tacit knowledge mainly spreads locally. On the other hand, firms can save travelling and other transaction costs if they locate close to each other in order to exploit tacit knowledge coming from other firms or institutions. This logic contains the conclusion that spatial concentration (or the spatial boundedness of knowledge spillovers) is only necessary in those industries where new knowledge is a critical competitive factor (Audretsch and Feldman, 1996), and where knowledge is basically tacit (Sorenson, 2005).

Another reason for spatial concentration of firms uses the arguments of trust and embeddedness. If we disregard market mediated knowledge transfers (i.e. when someone pays for knowledge), it turns out, that trust is inevitable for this kind of transactions. If one shares her knowledge with other actors, this transaction obviously erodes her competitive advantage (which clearly lies in knowledge of things which other actors do not know). In these circumstances it is not advantageous for the actor to share her knowledge unless she expects others to share their knowledge with her. This expectation, on the other hand, roots in mutual trust which requires a past relationship with positive experiences (embeddedness). To develop such trustful relationships, agents need frequent personal interaction which is clearly eased by spatial proximity. Therefore, we expect to see trustful relationships among agents who locate close to each other whereas less of these relationships among agents who locate farther away.

Those mentioned above, leads us to the notion of clusters. For firms, which heavily rely on new knowledge as the source of their competitive advantage it seems undoubtedly useful to locate close to each other, and, moreover, to establish strong linkages among each other in order to gain easy and immediate access to new knowledge. The resulting networks (or clusters) show tight cooperation, quick knowledge diffusion and a high level of innovativeness. Obviously, clustering tendencies and advantages from clustering differ among industries as these industries differ in the extent to which access to new knowledge is important, in the tacitness of knowledge used and whether the diffusion is mediated by knowledge sharing or knowledge broadcasting processes (Cowan, 2006).

In everyday language the word cluster is tightly related to innovativeness. According to those mentioned before, however, this is not that surprising. An industry in the early phases of its lifecycle relies heavily on newly generated knowledge (relative to other, more mature industries). On the other hand, in these industries the wider scale of technological opportunities rooting in the technological enfancy of the field leads to more innovations in a given period, thus contributing to a picture of dynamism and innovativeness. Moreover, in these cases knowledge is more tacit, first because codification is just in progress, and second because newly generated knowledge is inherently unstructured ant intuitive. Thus it seems logical that for an industry in its enfancy clustering proves considerable advantages contributing to dynamism and high innovativeness, while for more mature ones it is less important. Thus it seems clear that we observe dynamic, innovative clusters more frequently.

Recent literature on innovation emphasizes the role of heterogeneity and complementarity in the process of innovation. According to this a cluster becomes dynamic and innovative through the diversity of technologies, production processes employed and product variants produced by the firms in the industry (cluster). Using the terminology of the literature, we can say that firms operate on different knowledge bases (Pavitt, 1998; Nelson, 1998). The diversity of these knowledge bases give real innovation potential: the combination of different elements, recognizing complementarities reveal a wide space for innovations based on association. The strength of innovative clusters lies in the frequent interactions of diverse knowledge bases which is mediated by the increasing number of links between firms. Thus, advantages in diversity can be exploited rapidly.

On the other hand, it is understood that it is not heterogeneity itself that contributes to innovativeness, rather complementarities in knowledge. This leads to the recognition that innovation does not increase indefinitely as heterogeneity grows. Rather, there exists an inverted U-shaped relationship between the two (Nooteboom, 1999). Too little heterogeneity means that firms know mainly the same, thus there is no room for combining ideas: innovation activity is low. On the other hand, if heterogeneity is too high, firms do not even understand each other: they can not communicate effectively, thus innovation ceases. In our study we take this inverted U-shaped relationship between knowledge heterogeneity and innovativeness as given. For a detailed discussion on the topic see for example Cohen and Levinthal (1990), Nooteboom (2004), Wuyts et al. (2003), Cowan and Jonard (2007).

Clustering has another effect with respect to knowledge bases. Firms, who interact frequently, form joint research alliances, gradually loose their diversity as they learn from each other. After a while firms know the same as they know everything possible in the cluster, thus diversity disappears taking out the wind of innovation's sail (Cowan and Jonard, 2007). Thus we expect the cluster to have a special lifecycle in which the initial phase characterized by dynamism and innovativeness is followed by a mature and declining phase when innovation and dynamism erodes (Lengyel, 2002).

On the contrary, however, there is some evidence that innovation and heterogeneity can be sustained in the long run. These results are in line with the previous findings: if heterogeneity goes hand-in-hand with innovation, the two must be observed jointly, or none of it. Regarding the lifecycle detailed above, it seems that to maintain innovation, heterogeneity must be rebuilt in the cluster. This can be easily done by channelling new knowledge into the cluster from outside (Baum et al., 2003; Cowan, 2006). However, Knott (2003) builds a model in which heterogeneity and innovation is sustained inside the industry without extra-cluster linkages.

In this paper we focus our attention on the heterogeneity and innovativeness of clusters. We present a model in which firms in an industry innovate on their own or in alliance with each other. The success of an alliance depends on the diversity of knowledge bases of the allying firms. This diversity, in turn, is measured by their distance in knowledge space. With the analysis of this model we try to answer the questions of heterogeneity and innovation in clusters. Our second aim is to examine the characteristics of extra-cluster link formation and its effects on intra-cluster dynamics. How extra-cluster linkages are formed and to what extent can they affect the revival of innovativeness in the cluster. One of our main finding is that our model retains heterogeneity in the cluster while innovation follows the life-cycle pattern (decreases). This feature constitutes a useful extension of existing models of innovation in clusters, where heterogeneity and innovation go hand-in-hand, and thus gives a theoretical background for some empirical findings of persistent heterogeneity (Molina-Morales and Martínez-Fernández, 2004; Leiponen and Drejer, 2007).

The paper is structured as follows. In Section 2 we outline the model with all its important features. In Section 3 we present the results of the simulation of the model, and give an analysis of these results. In Section 4 we discuss our findings and Section 5 gives conclusions.

2 The Model

The industry is populated by N firms. Each firm is characterized by a knowledge-portfolio which covers both the breadth and depth of firms' knowledge bases (Prencipe, 2000). This portfolio of firm n is represented by the vector $\mathbf{k^n} = (k_1^n, k_2^n, \ldots, k_w^n)$, where w is the highest possible breadth of knowledge, and k_i^n represents firm n's knowledge level in technological (knowledge-) field i. The higher value of k_i^n, the deeper knowledge firm n has in field i. Of course, the elements of a firm's knowledge vector should be zeros, so we only assume that $k_i^n \geq 0$ for all $n \in (1,2,\ldots,n)$ and $i \in (1,2,\ldots w)$. Consequently, the more $k_i^n > 0$ firm n has, the broader is firm n's knowledge portfolio, i.e. it has competence in more technological fields.

In the simulations below, firms' knowledge portfolios are generated randomly at the outset, in two steps. First, every $i \in (1,2,\ldots w)$ technological field become part of firm n's portfolio with probability $\Pr(k_i^n > 0) = 1/2$, $\forall i, n$.[1] Second, if technological field i is part of firm n's portfolio, then a given k_i^n value is assigned to the firm, choosen randomly from the set $(1,2,\ldots,k_i^{\max})$. $k_i^{\max}$ stands for the technological frontier in technological field i. In turn, we can define the whole industry's technological frontier with the vector $\mathbf{k^{max}} = (k_1^{\max}, k_2^{\max}, \ldots k_w^{\max})$. This vector defines a w-dimensional cuboid in the w-space, the cubic capacity of which can be used as a measurement of the technological frontier. At the outset, $\mathbf{k^{max}}$ is given as $k_i^{\max} = k^{\max}$ for $\forall i$. Later on, the technological frontier evolves with firms' innovative activity.

A pair of firms is characterized by their distance in knowledge space. Although we measure knowledge along two different dimensions (i.e. breadth and depth), we can simply measure the similarity/dissimilarity of two firms by the euclidean distance of points $\mathbf{k^n}$ and $\mathbf{k^m}$ in the Euclidean w-space. Thus, the distance of firm n and firm m according to their knowledge bases is simply written as:

$$d_{n,m} = \sqrt{(k_1^n - k_1^m)^2 + (k_2^n - k_2^m)^2 + \ldots + (k_w^n - k_w^m)^2} = \sqrt{\sum_{i=1}^{w} (k_i^n - k_i^m)^2}$$

Measuring distance this way is reasonable, as mentioned in the Introduction. Firms can effectively communicate with each other if they share at least some technological fields they operate in. However, if firms are competent in exactly the same fields, they can still learn from each other if one firm knows more than the other, although which firm learns and which receives knowledge is predetermined in this case. On the other hand, for the effective communication it is required that firms be close in the depth of their knowledge as well, because otherwise one of them would be so advanced relative to the other, although in the same field, that their communication would break down.

2.1 Innovation

Innovation is modelled as a random process. Firms can innovate alone and in alliances with each other.

2.1.1 Separate innovation

[1] We analyze the effect of this probability on the initial knowledge portfolios of firms later.

When firms innovate alone, all they can do is to increase their knowledge level in one of their existing knowledge fields. In this case, firms innovate with probability p_0. If innovation occurs, one field is selected randomly from the firm's portfolio, and the knowledge level in this field is updated according to:

$$k_i^n(t+1) = k_i^n(t) + 1$$

where t is the time index. This formulation represents, that through innovation, firms move upwards on the knowledge ladder, deepening their knowledge in a given field. Assuming that the value of knowledge to the firm is represented by the knowledge level, we can simply calculate the value of the knowledge level for firm n at time t:

$$V^n(t) = \sum_{i=1}^{w} K_i^n(t)$$

When evaluating the value of the innovation, consider the expected value of firm n's knowledge base in the next period, which depends on the expected value of the individual knowledge fields:

$$E\left[V^n(t+1)\right] = \sum_{i=1}^{w} E\left[k_i^n(t+1)\right]$$

where E is the expected value operator. The expected knowledge level of technological field i in the next period can be written as follows:

$$E_i\left[k_i^n(t+1)\right] = \left[p_0\left(\frac{1}{w^n}\left(k_i^n(t)+1\right) + \frac{w^n-1}{w^n}k_i^n(t)\right) + (1-p_0)k_i^n(t)\right]\mathbf{1}_{k_i^n>0}$$

where $\mathbf{1}_{k_i^n>0}$ is the indicator function of $k_i^n > 0$ and w^n is the number of technological fields in which firm n is competent.[2]

Thus, the expected value of the knowledge base in period $(t+1)$ is:

$$E\left[V^n(t+1)\right] = \sum_{i,k_i^n(t)>0} p_0\left(\frac{1}{w^n}\left(k_i^n(t)+1\right) + \frac{w^n-1}{w^n}k_i^n(t)\right) + (1-p_0)k_i^n(t) = \frac{p_0}{w^n}\left(w^n \sum_{i,k_i^n(t)>0} k_i^n(t) + w^n\right) + (1-p_0)\sum_{i,k_i^n(t)>0} k_i^n(t) = \sum_i k_i^n(t) + p_0$$

where in the last equality we used the fact that $\sum_{i,k_i^n(t)>0} k_i^n(t) = \sum_i k_i^n(t)$ by definition. So we have a very simple form for the value of innovation in this case. The value of innovation can be simply characterized by the expected growth in the value of the knowledge base:

$$E\left[V^n(t+1)\right] - V^n(t) = p_0$$

Not too surprisingly, this tells us that if innovation increases one of firm's existing knowledge type by 1 unit, and if innovation occurs with probability p_0 than the expected value of this innovation is p_0.

2.1.2 Innovation in alliances

The other possibility for firms to innovate is to look for partners in the industry. However, if two firms form an alliance, not only innovation occurs, but they can learn from each other as well. We disregard this possibility for the time being, but incorporate it into the analysis in later sections.

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[2] w^n is simply calculated as $w^n = \sum_i \mathbf{1}_{k_i^n>0}$

Innovation between two firms takes the following form in our model. If firm n and m meet, they innovate with probability $p_{n,m}$, depending on their distance in knowledge-space: $p_{n,m} = f(d_{n,m})$. According to those mentioned in the Introduction, there is an optimal distance between firms, denoted by δ, where the probability of success is the highest. Getting farther away from this distance in either direction, the probability of success falls. Thus $f(d)$ is single-peaked at δ and symmetric around δ, being monotonically increasing if $d < \delta$ and monotonically decreasing if $d > \delta$. This formulation is borrowed from Cowand and Jonard (2007).

It seems straightforward that alliance firms will innovate in those areas where they both have competence. This can be acknowledged relatively easy by intuition. When R&D alliances form, the partners set the areas in which they will work together. However, a firm is not interested in choosing areas where the firm itself or the partner is not competent as it would radically lower the chances to successfully innovate. As research fields are narrowed this way, it becomes highly unlikely that the alliance will innovate at a field where either of the partners have no competence. However, we will relax this assumption later by allowing alliances to innovate on those fields where either of the allying firms have competence.

When deciding whether to form an alliance with another firm or not, firms must evaluate the expected value of joint innovation. As carried out for the autarchic innovation above, this can be done by simply calculating the expected value of firm n's knowledge base in period $(t+1)$ if it establishes an alliance with firm m. The main difference is two-fold. First, the porbability of successful innovation now is $p_{n,m}$. Second, innovation can occur only in those knowledge fields, where both firm n and firm m have competence. According to these, the expected value of firm n's knowledge base in technological field i if it cooperates with firm m, is:

$$E_i\left[k_i^n(t+1)\right] = \begin{cases} p_{n,m}\left[\frac{1}{w^{nm}}\left(k_i^n(t)+1\right) + \frac{w^{nm}-1}{w^{nm}}k_i^n(t)\right] + (1-p_{n,m})k_i^n(t); \forall i, k_i^n(t) > 0, k_i^m(t) > 0 \\ k_i^n(t) > 0; otherwise \end{cases}$$

where w^{nm} is the number of technological fields in which both firm n and firm m has competence.[3] From this, by summing through i, we get the expected value of firm n's knowledge base:

$$E\left[V^n(t+1)\right] = \sum_i k_i^n(t) + p_{n,m}$$

from which the value of joint innovation is:

$$E\left[V^n(t+1)\right] - V^n(t) = p_{n,m}$$

Of course, this method does not take into consideration, that in several cases firms do not have common technological fields. Instead of building this into the formulae above, we simply rule out this possibility by assuming that firms do not form alliances if they have no common technological fields. In this case, the equations above are correct, as if firm n and firm m do not have common fields, they do not evaluate the value of their joint work, as it is apparently zero.

2.2 Network Formation

Given these results, we can see, which firms will form an alliance. Consider, that the maintenance cost of a partnership is c in each period.[4] In this case, an alliance between firm n and firm m will form, if

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[3] Formally, it can be written that $w^{nm} = \sum_i \mathbf{1}_{k_i^n > 0, k_i^m > 0}$

[4] Of course, it is a simplification that the cost of maintaining a relationship is independent of the number of these relationships: these costs may increase as the number of links grow.

$$p_{n,m} > c$$

Therefore, the formation of a link is simply the function of the distance of the two firms considered. Holding c constant, the number of links a firm has is only dependent on the average distance between it and the other firms. This distance, in turn, depends on the parameters of the outset of the model, namely $k_{\max}$, w and p_e. As $d_{n,m} = d_{m,n}$ by definition, $p_{n,m} = p_{m,n}$. This means that if a partnership is profitable for firm n, it is also profitable for firm m. So links will be stable in the sense that all alliances that form is benefitial to both partners, thus they are interested in keeping it alive at least until the next period when knowledge bases change, thus firms' distances change as well.

In the simulations below, following Cowan and Jonard (2007), we use the simplest form of $f(d_{n,m})$:

$$p_{n,m} = 2p_0\left(1 - \frac{|d_{n,m} - \delta|}{2\rho}\right)$$

where ρ measures the base width of the inverted U, i.e. the larger is ρ the more firms are suitable as partners according to their distance from firm n. δ is the optimal distance for innovation, as mentioned earlier.

3 Simulations

3.1 Initial Knowledge Portfolios and the Structure of the Network

First we check how the parameters of the model influence the properties of the initial industry, i.e. the characteristics of the network that forms according to different values of the parameters. The analysis was done by Monte Carlos simulations. We made 1000 runs of the simulations with each time randomly generated input parameters. The simulations were made only for the first period, i.e. we generated knowledge portfolios and analysed the emerging network structure and industry characteristics.

We use four outcome measures that carry some information on the characteristics of the network and the industry. These are the following.

Heterogeneity. Heterogeneity seems to be an important factor in defining innovativeness (Cowan and Jonard, 2007; Knott, 2003). However, the heterogeneity of a population can be measured in different ways. In our context heterogeneity means diversity in the knowledge bases of the firms. To measure heterogeneity we will use statistical entropy. Entropy measures the skewness of the distribution of a population in space: if the agents are similar, entropy is low, if they differ, entropy is high (Frenken, 2004). In our analysis entropy measures how much firms are scattered throughout the knowledge space. Although interesting in the initial distribution as well, the heterogeneity of the industry becomes more important in the dynamic setting.

Innovativeness. As our main focus is on innovation activity, we measure how innovative firms are in the industry. We do this by simply counting the number of innovations appeared in each run (both joint and own innovations of the firms).

Average degree of a network. Along innovativeness and heterogeneity we are interested in the features of the underlying industrial network. For this as a first measure we use the average degree of the network, i.e. the average number of links firms have.

Clustering. As the main focus of our examination is the evolution of clusters, we use the so called clustering coefficient as an output variable. The clustering coefficient measures how much a network is clustered, i.e. how much 'one's friends are friends of each other' (Cowan, 2005). As this measure also consists of counting links in a network, its value correlates with that of average degree. However, the clustering coefficient reveals some additional information on the structure of the network than simply counting the links between agents.

The analysis was done by regressing the output variables on the input parameters. The regression statistics can be found in the Appendix.

3.1.1 Heterogeneity

We made different regressions by including the parameters into the model one by one.[5] The main conclusion is that although N, w and p_e all have significant effect on entropy, the inclusion of w and p_e into the regression model does not affect significantly its fit (measured by adjusted R^2). The highest part of the variance in heterogeneity is accounted for by the number of firms in the industry.

However, this result is straightforward as the entropy statistics is not independent of the sample size. To overcome this problem, we normalize the entropy measure in order to eliminate this built-in effect.[6] After controlling for this effect, and using relative entropy (entropy divided by maximum entropy) as a measure of heterogeneity, we find that the number of firms does not really affect the heterogeneity of knowledge portfolios, while w and p_e have positive significant effects.

When N was eliminated from the determining factors,[7] the fit of the model heavily falls to 0.21, which means that although some effect of w and p_e can be detected, the reasonable part of the variation in heterogeneity remains unexplained by the parameters of the model, it is due simply to stochastic elements in the simulation.[8]

The results above are somewhat expectable, as firms' knowledge portfolios are selected randomly, thus we expect the initial landscape of the industry being the most heterogeneous possible. This is proved by the data as well, as the relative entropy of the industries is over 0.999 in more than 85% of the simulated cases. To have a benchmark for the analysis above, we run the regression on a subset of our dataset excluding those cases where the relative entropy exceeds 0.999.

As it was expected, the fit of the model increases, but only slightly, and the significant independent variables remain unchanged.[9] So, after controlling for the randomness of the knowledge portfolio generating process, we can conclude that the number of relevant technological fields (w) and the probability with which these fields belong to a firm's knowledge portfolio (p_e) has positive, significant effect on the heterogeneity of firms, however these factors explain a relatively small amount of the variance in the entropy.

3.1.2 Innovation

Regarding innovation, the results of the regression analysis show, that almost all parameters of the model have significant impact on the innovation potential of the initially emerging network. More firms (increasing N) obviously lead to more innovation: first, because own innovation increases (holding p_0 fixed); second, because the increase in the number of possible links increases the expected number of joint innovations. The effect of the probability of innovation (p_0) has a similar, intuitively clear effect on the number of innovations. The effect of w and $k_{\max}$ is significant and negative, whereas p_e has no significant effect.[10]

The reason why δ has no significant effect is the following. An increase in δ can either increase or decrease the number of innovations in the industry. Which happens depends on the average distance of firms. If δ moves towards average distance (meaning that more firms get closer to the optimal distance), innovation shall increase. In the opposite case, when δ moves away from the average distance (i.e. firms get farther away from optimal), innovation decreases. Thus, the sign of δ depends on the relative value of average distance and optimal distance, for which we can not control in our dataset. The effect of ρ is positive and significant, supporting the simple intuition that a wider

[5] For the statistics see Table A1 in the Appendix.

[6] The effect of sample size in entropy is hidden in the fact that given the sample size N, the level of entropy can vary between 0 and $\ln N$. If the network is heterogeneous, i.e. the entropy is close to its maximum, this drives the regression model to measure significant effect of N on heterogeneity, although the underlying networks are qualitatively the same with respect to heterogeneity as they are all close to the maximum.

[7] For the statistics see Table A2.

[8] The inclusion of further parameters (δ, ρ, etc. is unuseful as entropy depends solely on the distribution of knowledge portfolios in the knowledge space and has nothing to do with network structure.

[9] For numerical results see Table A3.

[10] See the results in Table A4.

suitable distance from optimal knowledge distance gives more place to alliances thus leading to more innovation, holding other things constant.

We must highlight, however, that much of the simulated cases (about 75%) led to empty networks, where firms innovate only on their own. This, in turn, leads to decreased R^2 because in the empty network whether firms innovate or not (thus the number of innovations) depend only on p_0. To control for this huge portion of the sample showing uninteresting results, we run the regression on the subset of the sample where the resulting network was not empty.[11] The results for this analysis are more convincing: the adjusted R^2 increases to 0.339. The effect of the parameters do not change considerably, but their significance decrease by a small extent.[12]

In these experiments all parameters were selected independently in the beginning of each run. This resulted in 75% of the cases being empty networks. There is a way, however at least in principle to reduce this amount by imposing restrictions on some parameters. We reproduced the simulation with the (random) values of δ and ρ restricted to some convenient intervals, depending on $k_{\max}$.[13] The results change only slightly. Due to a higher portion of the emerging networks being not empty, the explanatory power of the new regression analysis somewhat increased, but not considerably. On the other hand, the restrictions created multicollinearity among independent variables which caused problems in the interpretation. This modification of the simulation thus seems unreasonable.

3.1.3 Degree and clustering

We made the same analysis as before for the average degree and clustering coefficient of the networks. We discuss the two measures together as they are closely interconnected as mentioned before. As the full sample of industries contain 75% of empty networks with zero degree and clustering, we exclude these cases from our analysis, so we work with the subsample of those cases where the degree exceeds zero (just like in the case of innovations before). The explanatory power of the model increases if the empty networks are excluded from the sample.[14]

The only major difference between the two regressions is that N has a positive effect on degree, while it does not affect clustering. This feature comes from the fact that the possible number of links a firm can have increases with the number of firms, thus degree is positively influenced by N, ceteris paribus. The clustering coefficient, on the other hand, is normalized by the number of agents, i.e. it measures the ratio of 'triangle-closing' links in one's neighbourhood to that of all possible links in that neighbourhood (Cowan and Jonard, 2007).

Average degree is only determined by N (as mentioned above), $k_{\max}$ and ρ. If $k_{\max}$ increases, degree decreases and the opposite is true in the case of ρ. The reasons are similar to those mentioned in the case of innovations, as degree and innovations depend on the number of links formed in the industry. In the case of clustering, however, p_e and δ join the significant independent variables, however their significance is not that high. On the other hand it is also visible that the fit of the model is in practice determined by including ρ among the explanatory variables (the adjusted R^2 jumps from the invisible 0.043 to 0.32).[15] This shows us that although the coefficients of other parameters are proved significant, their contribution to the variation in degree and clustering is not that important.

This result seems straightforwrad. Although $k_{\max}$, p_e and δ are shown to have an effect on the two output measures, this effect is not intuitive. Changing the first two parameters leads to a change in the size of the knowledge space, thus leading to a change in average distance between firms. Whether

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[11] A network was considered 'not empty' if the average degree exceeded zero, i.e. if at least one link existed among firms.

[12] See the results in Table A5

[13] Our method rests on the observation that once $k_{\max}$ is determined, through average distance between firms it restricts those values of δ which can lead to not-empty networks. It can be easily proved that if δ exceeds $\sqrt{2}k_{\max}$, than no links can be formed with ρ assumed to equal zero. Thus we restricted the value of δ to the interval $(0;\sqrt{2}k_{\max})$. On the other hand, ρ is restricted to the interval $(0;\min(\delta;\sqrt{2}k_{\max}-\delta)$. On these intervals δ and ρ were selected randomly.

[14] Detailed results are presented in Tables A6 and A7.

[15] This is true in both the degree and clustering regressions.

this change leads to more links and clustering, depends on the relative value of average distance to optimal distance for innovation (see the analysis in the case of innovations before). The case of parameter δ is the opposite, as the sign of its effect is determined by the average distance. Parameter ρ, on the other hand, defines the area around optimal distance in which joint innovation can be profitable. If this area increases, regardless of average distance among firms, more pairs of firms will find profitable to form an alliance, and degree and clustering will increase.

This analysis was carried out also with the aforementioned restricted intervals of δ and ρ. The results for this are the same as in the case of innovations: increasing the adjusted R^2 by a small extent, but creating controversies through multicolleration.

3.1.4 Correlation between output variables

After analysing the effect of different parameters on the characteristics of the emerging industry network, it seems useful to have a glance on the correlation between the output variables, i.e. relative entropy, innovation, degree and clustering. Table 1 shows the results for the correlation analysis.

Table 1 Correlation between Output Variables

		deg	ino	cls	relent
deg	Pearson Correlation	1	,646(**)	,789(**)	-,078
	Sig. (2-tailed)		,000	,000	,451
	N	96	96	96	96
ino	Pearson Correlation	,646(**)	1	,490(**)	,016
	Sig. (2-tailed)	,000		,000	,880
	N	96	96	96	96
cls	Pearson Correlation	,789(**)	,490(**)	1	-,155
	Sig. (2-tailed)	,000	,000		,131
	N	96	96	96	96
relent	Pearson Correlation	-,078	,016	-,155	1
	Sig. (2-tailed)	,451	,880	,131	
	N	96	96	96	96

** Correlation is significant at the 0.01 level (2-tailed).

The correlation coefficients were calculated for those records in the dataset, where entropy is not full, and the network is not an empty one.[16] The results are as follows. We find positive, significant correlation among innovation, clustering and degree. This comes from two facts. First, that clustering and degree correlate with each other by definition, as mentioned above, second, that more links lead to more innovation holding fixed the probability of innovation. On the other hand, we find no significant correlation between relative entropy and any of the other variables. This result is the same if we use absolute entropy instead of relative.

As a benchmark, we calculated the same coefficients for the full dataset, i.e. all empty networks and full entropies included. On this dataset we find significant correlation between relative entropy and the other output variables, however these correlation coefficients are much smaller than those of the others, and are negative, showing that more heterogeneity leads to less links among firms and less innovation. This results roots in the obvious fact that much of the observations include full entropy and zero degree and clustering at the same time which leads the correlation statistics to state that high entropy matches with small degree and innovation.

However, the results of entropy having no significant effect on innovation can be easily understood in our setting. Our model of innovation does not state that heterogeneity is unconditionally favourable for innovation, rather that there exists an inverted U-shaped relationship. Too much homogeneity leads to less innovation as firms have very few things to learn from each other. On the other hand, too much heterogeneity leads to less innovation as well, because firms can not efficiently communicate with each other, thus less links form in the industry.

3.2 Dynamic Analysis

Although the characteristics of the initially emerging network give some important insight into the mechanisms of the model-industry, we are mainly interested in the dynamics of the networks, i.e. how it

[16] This meant 115 observations out of the simulated 1000.

evolves over time through innovation and the consequent evolution of knowledge bases. As outlined in Section 2, firms innovate either on their own or in alliance with other firms. Through innovation, firms' knowledge bases change as well, i.e. they know more in the consecuting period. This change in the knowledge base can lead to different network structures than before. In this section we analyse this dynamics, using the formerly introduced output variables to evaluate the outcomes.

The method is basically the same: we run a Monte Carlo simulation with 1000 independent experiments. In each one the input parameters were selected randomly, and than after generating the initial knowledge portfolios and the emerging network, the alliance-formation and innovation process were iterated 300 times consecutively (i.e. we had 300 periods). The output variables were calculated for each period, and we use these values to analyse the dynamics.

3.2.1 Stable vs. evolving networks

As a first glance on the data one realizes that part of the experiments lead to a stable network, i.e. the output variables do not change over the 300 periods. These networks, however, largely correspond to empty networks: out of the 1000 experiments we found only 30 cases where an initially not empty network remained stable, whereas only 22 cases occured where an initially empty network changed over time. These together give only 5% of the experiments. The remaining majority of the cases show either initially empty networks remaining empty or initially not empty networks evolving over time.

We took those 948 samples where either change occured or the networks remained empty and used a binary logistic regression model to see which paramerters of the model affect the evolution.[17] The detailed results are presented in Table A8.

We run the regression by including the parameters as independent variables one by one. The explanatory power (as measured by the Nagelkerke R^2) gradually increases as the new parameters are included in the model, the best fit is generated by including all parameters. The best fit corresponds to a Nagelkerke R^2 of 0.549. The results show that all parameters have significant effect on the probability that a network evolves or not, except p_0. The number of firms has a positive effect, showing that more firms lead to higher probability of a change in the network structure. The parameters determining the initial knowledge portfolios (i.e. w, $k_{\max}$ and p_e) all have negative effect on this probability. These parameters affect initial knowledge portfolios and through this effect they determine the initially emerging network. The fact that p_0 has no significant states that the overall probability with which innovation occurs does not affect the evolution of networks: should it be higher or lower, the dynamics of the underlying network do not change considerably. Parameters δ and ρ have a significant positive effect on the evolution: higher their values, higher the probability of getting an evolving network. These results can also be interpreted as the effect of the parameters affecting the initially emerging networks being empty or not.[18]

3.2.2 The evolution of the networks

Our main interest is in the subset of those networks where some change in the network structure can be detected. In these cases we analysed the overall direction of the change. For this reason we calculated the difference of the output variables between periods and averaged them over the 300 periods to gain a measure of the direction of change. If this exceeds zero, we conclude that the given variable increases through time and vice versa.

Entropy

First, we look at the evolution of the entropy of the networks. For the analysis hereon we use the relative entropy measure. Initially more than 80% of the networks show full entropy, and after 10 periods this share increases to 97%. At the end of the 300 periods more than 98% of the networks (532 out of 542) is totally heterogeneous according to entropy. The average change in relative entropy, although very low, is positive or zero in each case. Thus we can conclude that in our model, irrespective of the parameters, heterogeneity does not decrease through time with the evolution of the networks. This is reassured by our simple regression analysis in which we regressed average change in relative entropy on the model parameters. The results indicate that the highest adjusted R^2 is 0.073, and all parameters

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[17] We used only average degree, entropy and clustering to evaluate the stability of the network, as innovation is not a characteristic of the network itself, and as a random process, it changes from period to period without an underlying change in the network structure.

[18] As a benchmark, we have the same results by running the regression on the initial portfolios.

except w and p_e were found insignificant (see results in table A9). We come back to this result later on.

Innovation

Regarding innovation, we found that in 493 out of the 541 observed cases innovation decreases over time. The average of the average decreases is -16.3. In the remaining 48 cases innovation increases on the average, and the average is only 2.1. This leads to the conclusion that innovation is decreasing in our model irrespective of the model parameters, increasing innovation being an exception rather than a rule.[19] A further analysis was made to detect the effect of different parameters on the extent of the average decrease in innovation. We regressed the average change in innovation on the model parameters, and got a model with adjusted R^2 equal to 0.42. All the parameters except p_e were found to have significant effect on the change in innovation. This is not surprising, however, as innovation itself is determined by these parameters (see the analysis of the initial portfolios), so one must expect the absolute value of change in innovation to vary with the model parameters. To rule this effect out, we normalized the number of innovation with innovation in the first period. This way we got a measure of relative change in innovation. The average of the 300th period is 0.418 which states that on average networks innovate only 42% of their first-period innovations in the 300th period,[20] which clearly shows that innovation decreases over time. To evaluate whether model parameters really have an effect on the change in innovation, we regressed these 300th period innovation ratios on the parameters. As a result, we got a very loose relationship with adjusted R^2 equal to 0.002 and with highly insignificant coefficients (see the results in table A10). These results show that the extent of the decrease in innovation is independent of the model parameters, the variation in the 300th period relative innovations is only due to chance.

Degree and clustering

In the case of degree and clustering, we followed the method described above in the case of innovations: we calculated relative degree and clustering as the ratio of its actual value to that of the first period. In the case of degree we found that on average the 300th period degree is 28.3% of the first period value, whereas in the case of clustering this ratio is 25%. There are 15 and 12 cases out of 541 in the case of degree and clustering respectively where the 300th period value is higher than the first period one. The average of these cases, however is more considerable than in the case of innovation: 426% in degree and 177% in clustering, and we can not convincingly detect outliers here as half of the cases are of that kind. However, the results clearly show, that apart from some 2% of the simulated cases the structure of the networks gradually dissolve as time passes by. In addition we carried out a scrupulous regression analysis to see the effect of parameters on the evolution of degree and clustering, and in both cases we found small R^2 with correspondingly insignificant coefficients (see tables A11 and A12).

3.2.3 Innovation and heterogeneity

As a final issue we can give a glance to the correlation between output-variables, i.e. if there is a connection between the evolution of degree and entropy, etc.

The findings are as follows (see Table 2). Significant correlation can be detected between clustering and degree, but it is not surprising, as mentioned above with regard to the analysis of the initial networks. There is a very strong, significant correlation, however, between degree and innovation and clustering and innovation. This means that if the network grows, innovation increases, whereas if the network breaks apart, innovation decreases. This result is also not that surprising as already mentioned in the previous sections.

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[19] However, this number is mainly due to three outliers with 13.7, 63.4 and 9.9 as average increase. If we exclude these from our sample, the average decreases to 0.3 which is not significantly different from zero. Moreover, in 2 out of the 3 outliers it can be shown that innovation first increases, but eventually starts to decrease, thus it seems that in these cases a longer experimental horizon would have led to a decreasing average change in innovation.

[20] With the three outliers excluded from the dataset.

Table 2 Correlation of the Direction of Change in Output Variables

		deg_avgch	ino_avgch	cls_avgch	ent_avgch	relent_avgch
deg_avgch	Pearson Correlation	1	,763(**)	,692(**)	,024	,039
	Sig. (2-tailed)		,000	,000	,573	,359
	N	541	541	541	541	541
ino_avgch	Pearson Correlation	,763(**)	1	,445(**)	,005	,023
	Sig. (2-tailed)	,000		,000	,911	,588
	N	541	541	541	541	541
cls_avgch	Pearson Correlation	,692(**)	,445(**)	1	,107(*)	,128(**)
	Sig. (2-tailed)	,000	,000		,013	,003
	N	541	541	541	541	541
ent_avgch	Pearson Correlation	,024	,005	,107(*)	1	,989(**)
	Sig. (2-tailed)	,573	,911	,013		,000
	N	541	541	541	541	541
relent_avgch	Pearson Correlation	,039	,023	,128(**)	,989(**)	1
	Sig. (2-tailed)	,359	,588	,003	,000	
	N	541	541	541	541	541

** Correlation is significant at the 0.01 level (2-tailed).
* Correlation is significant at the 0.05 level (2-tailed).

However, we find no significant relationship between entropy (heterogeneity) and the other output variables (neither in the case of relative, nor absolute entropy). This is an important result, showing that heterogeneity is persistent, but innovation and clustering ceases. This is interesting in the dynamic setting, as here firms have enough time to converge to each other in the knowledge space through innovation, but they do not. Some firms break ahead, permanently pushing out the technological frontier, while others try to keep up with them. Heterogeneity does not decrease but rather increase. This result is in line with some recent research in the industrial organization and innovation literature (see e.g. Knott, 2003; Molina-Morales and Martínez-Fernández, 2004; Leiponen and Drejer, 2007).

This latter result seems consistent with the findings of Cowan and Jonard (2007) who conclude in a basically similar setting that all networks break up eventually, however, the reason for this in their model is that firms become too homogeneous and this hampers alliance-formation thus leading to the fall of a cluster. In our model networks fall, their innovation potential decreases but heterogeneity is persistent.

Our findings also shade the results of Knott (2003), where persistent heterogeneity is connected to sustainable innovation. Our model demonstrates, that using a reasonable setting for innovation in knowledge-networks, innovation eventually decreases whereas heterogeneity is persistent. This does not mean, however that the former approaches are of no value. Rather, we see our model as an addition to those mentioned above, contributing to a typology of cluster-evolution.

This typology differentiates between clusters with respect to the evolution of the heterogeneity of cluster firms and that of the innovation potential of the cluster. With two classes along both dimensions we can distinguish among four types of cluster-evolution.

The first category can be labelled as 'dynamic', showing both sustainable innovation and persistent heterogeneity. The model of Knott (2003) fits to this type.

The opposite category is that of decreasing heterogeneity (firms becoming homogenous) and decreasing innovation. This type may be named 'declining' and the model of Cowan and Jonard (2007) fits to this one.

Our model shows persistent heterogeneity but decreasing innovation, leading to a 'fragmented' cluster with different firms but with small innovative potential.

The fourth version is the case of homogeneous firms and sustainable innovation, which could be labelled 'competitive', but we have found no suitable model for this type, although the situation is not impossible in principle.

Table 3 Typology of Cluster Evolution Regarding Heterogeneity and Innovation

	Sustainable Innovation	Decreasing Innovation
Persistent Heterogeneity	**Dynamic cluster** *Knott (2004)*	**Fragmented cluster** *This model*
Decreasing Heterogeneity	**Competitive cluster** *?*	**Declining cluster** *Cowan&Jonard (2007)*

3.3 Dynamics with Learning

In the experiments above firms did not learn from each other, but innovated in those areas where both partners had competence. The results showed that heterogeneity does not decrease in the long run. However, if we let firms learn from each other, heterogeneity may fall. Learning may be viewed as firms integrating new knowledge areas in their portfolios. The selection of the new areas depend on the knowledge portfolio of the partners: firms do not gain new knowledge fields 'from the air' but learn it from others through alliances.

In order to examine the implication of learning in our model, we proceeded two ways. The first solution can be regarded as 'learning through innovation'. In this case a pair of firms (alliance formation depending on cognitive distance henceforward) can innovate in all fields in which either of them has competence. If the knowledge field where the innovation occurs belongs to only one firm's portfolio, the other firm will integrate it into its portfolio as well, so it will learn from the other firm something new. However, this method only accounts for learning when innovation occurs as well and disregards the possibility of autonomous learning in the sense that firms may learn from each other without successful innovation. This concept characterizes our second way of examining learning in the model. We modify the model by allowing firms to 'innovate' also in those situations when the joint innovation is not successful.[21] In these cases one of the firms integrate a new knowledge field into its portfolio, which is part of the partner's portfolio.

3.3.1 Learning through joint innovation

Analyzing the results of the simulations when learning through innovation may occur, one find few differences compared to the results detailed above. Nearly all of the numbers change insignificantly. Out of the 1000 experiments now 421 showed empty networks throughout the 300 periods and there were 510 evolving networks with initially not empty ones. We found 53 cases when an initially not empty network remained unchanged and only 16 cases when an initially empty network changed. These together give only 7% of the experiments.

The analysis of the effect of our model parameters on the evolution gives indistinguishably the same results as above. The number of firms affect positively, the parameters of the initial knowledge portfolios as well as δ and ρ affect negatively the probability that a network evolves over time. The explanatory power of the regression is higher than 0.5.

Regarding the evolution of the different output measures, we found the following results. At the end of the 300 periods more than 98% of the networks showed full entropy, and the average change in relative entropy is, although very low, positive or zero. As a consequence, the model parameters do not affect significantly the evolution of relative entropy.

The results for relative innovation are a bit different from the previous ones. We found that out of the 510 cases when the underlying network had changed, in 456 (which is 89.4%) innovation decreases over time. However, calculating the relative innovations in the 300th period, we concluded that the average level of innovation was 74% of that of the first period which is significantly higher than the 42% of the model without learning. This result suggests that when learning through innovation is allowed, although the tendency of innovation does not change (it is decreasing over time), the change becomes slower: networks do not loose that much from their innovative potential, given the same time

[21] Note, that technically we can not distinguish between innovation and learning in our model. Learning means that a firm has new knowledge which is the same as innovation. The difference is that learning means new knowledge only to the firm itself. (On the other hand, innovation is not necessarily new to the industry as a whole: alliances may innovate something which is already known by others.)

period.[22] The regression analysis of the model parameters again shows high explanatory power with insignificant parameters.

The same methods were applied for degree and clustering. In the case of degree we found that the degree of the 300th period was on average only 25.8% of that of the first period which is not significantly different from the 28.3% of the learningless analysis. The case of clustering is a bit different: the 300th period clustering coefficient was on average 43% of the first period one, whereas in the former case it was 25%.

This suggests that in the case of learning through innovation the degree of the network changes in the same manner as without such learning, but clustering decreases more slowly. This result, moreover, reflects the fact that although degree and clustering are closely interconnected, they are not the same. Cowan (2006) highlights that the most effective networks regarding knowledge diffusion seem to be the so called 'small worlds' which show short average distance and high clustering. Connecting them to these findings, our results imply that networks tend to be more clustered in the long run if learning is present. On the other hand, these findings put the results of Cowan (2006) into a dynamic setting: it seems that learning is facilitated by small worlds with high clustering, but in our dynamic model the inclusion of learning maintains this favourable environment in the long run.

On the other hand, there are 33 and 35 cases out of the 510 when degree and clustering respectively is higher in the 300th period than in the first one. The average increase is 472.6% with respect to clustering and 1089% with respect to degree which is again more considerable than in the case of no learning. This suggests that when learning through innovation is accounted for, an increase in innovation, degree and clustering is more likely than in the case of no learning. These cases, however, give only 6-7% of the experiments which still remain inconsiderable. The regression analysis for the effect of the model parameters on degree and clustering give small R^2 s and not too significant coefficients.

3.3.2 Autonomous learning

The experiments with autonomous learning give the same results as before. Out of the 1000 experiments we found 64 where an initially not empty network remained unchanged over time and only 16 where an initially empty networked changed. Out of the remaining 920 cases 536 showed change, the others were those with an initially empty networks remaining empty throughout the 300 periods.

The evolution of entropy in the autonomous learning experiments is the same as before: more than 80% of the networks showed full entropy in the beginning and entropy increased as time passed by. In the 300th period more than 98% of the networks was fully inordinate.

Innovation was decreasing on average, with innovative activity in the last period being 62% that of the first period. This is again significantly higher than the 41% of the learningless experiments, but not too different from the 74% of the learning through innovation case. As a conclusion we might state that learning (irrespective of its form) contributes positively to the long run innovativeness of networks, however we do not dare to differentiate between autonomous learning and learning through innovation.

As in the learning through innovation case, the clustering coefficient in the 300th period is 43% of that of the first period, which is higher compared to the learningless experiments. However, now the 300th period degree also seems to be higher 39% compared to a 28%. This difference, however, is not too impressive. On the other hand the degree and clustering coefficient of the last period is higher than the first period value in 33 and 51 cases respectively, which shows that including learning in the model increases the possibility of finding a growing network after 300 periods of run.

In this set of experiments, these 84 cases constitute more than 15% of the 536 cases, so it is worth examining which parameters of our model have an effect on an expanding network. To see this, we run a simple binary logistic regression with expanding vs. contracting networks as the dependent variable and the model parameters as regressors. The analysis gives a Nagelkerke R^2 of 0.217, which is not too convincing. As for the parameters, only $k_{\max}$, p_0 and ρ were found significant.

4 Conclusions

In this section we summarize our results gained from the simulations described above and provide some conclusions.

Our first observations were drafted according to the parameters of the model and their effect on the characteristics of the initially emerging network. The main conclusion was that the model parameters do

[22] It is important to note, that the number of innovations were calculated as before: learning do not count as innovation, although we have emphasized that technically they are equivalent.

not explain considerably the network structure, innovation and heterogeneity, although some correlations were found. This result is maintained after controlling for empty networks in the initial set. These correlations, on the other hand, are as expected.

The analysis of the initially emerging networks reveals interesting results for the correlation of different network characteristics. We find no significant correlation between the heterogeneity of the network and innovation. This clearly mirrors our supposition that heterogeneity is not per se contributive to innovativeness, but there exists an inverted U-shaped relationship instead.

The main focus of our model was on the dynamic evolution of networks through time. This analysis has led to interesting results. As a first intuition one would expect that as time passes by, firms become more similar through joint innovation. Our results, however, show the opposite. Heterogeneity does not decrease but rather increase in our setting. Increasing heterogeneity on the other hand do not lead to increasing innovation as compared to the results of other studies. In our model innovation together with clustering and degree decreases over time. That is, although heterogeneity is persistent, networks dissolve over time causing innovation to fall in the industry.

These results have led us to set up a typology of cluster dynamics where we differentiate the evolution of networks (clusters) along two dimensions. The first is whether heterogeneity is persistent or falling, the other is whether innovation is maintained or decreasing in the long run. The literature provides examples for persistent heterogeneity with maintained innovation as well as for decreasing heterogeneity and decreasing innovation. Our model provides example for persistent heterogeneity with decreasing innovation suggesting that heterogeneity per se is not a sufficient condition for long run innovativeness.

We examined further the evolution of networks by including learning in out setting. As a first intuition, this gives a 'tackle' to our networks, meaning that firms are becoming more homogeneous, as not only joint innovation brings them closer as before, but learning as well. We examined two kinds of learning: learning through innovation and autonomous learning. In both cases learning is conditional on alliances. The inclusion of learning in the simulations gives the impression that learning somewhat contributes to innovation and clustering, however it does not cancel their overall decreasing trend. Rather, when learning is present, the rate of decline in innovation, degree and clustering is smaller but is not nonnegative. On the other hand, heterogeneity is still non-decreasing in either case of learning. Whether learning happens through innovation or autonomously, does not affect considerably these results.

In this paper we tried to analyse extensively the presented model, however lot of interesting questions remained untouched. One is how the evolution of networks changes if the cost of maintaining an alliance is not independent of the number of these relationships. One would expect that the 'dynamism' of innovation and network-formation falls in this case. The other interesting question is the presence of multi-agent alliances. These are all important lines to further analyse this setting.

The main issue in this line, however, is decreasing innovation and the possibility of refreshing innovativeness in an otherwise declining cluster. It is important to see how extra-cluster linkages form and how these links can channel extra-cluster knowledge into the network. Our findings suggest that different mechanisms must work in the case of a declining cluster (with decreasing heterogeneity and innovation) and a fragmented cluster (persistent heterogeneity and decreasing innovation) of this paper. We find this question as a perspective line for further research.

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Appendix

Table A1 Results of the regression analysis for absolute entropy

Model Summary

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate
1	,752(a)	,566	,565	,64523
2	,764(b)	,583	,582	,63225
3	,764(c)	,583	,582	,63253
4	,791(d)	,626	,625	,59942

a Predictors: (Constant), N
b Predictors: (Constant), N, w
c Predictors: (Constant), N, w, kmax
d Predictors: (Constant), N, w, kmax, pe

Coefficients(a)

Model		Unstandardized Coefficients		Standardized Coefficients		
		B	Std. Error	Beta	t	Sig.
1	(Constant)	2,219	,042		52,353	,000
	N	,026	,001	,752	36,041	,000
2	(Constant)	1,977	,056		35,467	,000
	N	,026	,001	,758	37,052	,000
	w	,005	,001	,133	6,511	,000
3	(Constant)	1,965	,065		30,155	,000
	N	,026	,001	,758	37,033	,000
	w	,005	,001	,133	6,497	,000
	kmax	,000	,001	,007	,351	,726
4	(Constant)	1,597	,071		22,582	,000
	N	,026	,001	,763	39,286	,000
	w	,005	,001	,136	7,000	,000
	kmax	,000	,001	,010	,540	,589
	pe	,684	,064	,207	10,681	,000

a Dependent Variable: ent

Table A2 Results of the regression analysis for relative entropy

Model Summary

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate
1	,031(a)	,001	,000	,14257
2	,237(b)	,056	,054	,13864
3	,238(c)	,056	,054	,13869
4	,449(d)	,202	,198	,12765

a Predictors: (Constant), N
b Predictors: (Constant), N, w
c Predictors: (Constant), N, w, kmax
d Predictors: (Constant), N, w, kmax, pe

Coefficients(a)

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.
		B	Std. Error	Beta		
1	(Constant)	,970	,009		103,611	,000
	N	,000	,000	-,031	-,969	,333
2	(Constant)	,908	,012		74,295	,000
	N	-9,61E-005	,000	-,019	-,628	,530
	w	,001	,000	,235	7,634	,000
3	(Constant)	,904	,014		63,250	,000
	N	-9,65E-005	,000	-,019	-,630	,529
	w	,001	,000	,235	7,615	,000
	kmax	8,79E-005	,000	,018	,586	,558
4	(Constant)	,805	,015		53,458	,000
	N	-5,84E-005	,000	-,012	-,414	,679
	w	,001	,000	,240	8,455	,000
	kmax	,000	,000	,024	,851	,395
	pe	,183	,014	,381	13,447	,000

a Dependent Variable: relent

Table A3 Results of the regression analysis for relative entropy with maximum entropy cases excluded

Model Summary

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate
1	,079(a)	,006	-,002	,29014
2	,093(b)	,009	-,007	,29093
3	,109(c)	,012	-,012	,29162
4	,496(d)	,246	,222	,25574

a Predictors: (Constant), N
b Predictors: (Constant), N, w
c Predictors: (Constant), N, w, kmax
d Predictors: (Constant), N, w, kmax, pe

Coefficients(a)

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.
		B	Std. Error	Beta		
1	(Constant)	,659	,061		10,872	,000
	N	,001	,001	,079	,894	,373
2	(Constant)	,647	,065		9,971	,000
	N	,001	,001	,078	,875	,383
	w	,001	,001	,049	,558	,578
3	(Constant)	,623	,075		8,253	,000
	N	,001	,001	,076	,859	,392
	w	,000	,001	,042	,464	,643
	kmax	,001	,001	,057	,633	,528
4	(Constant)	,445	,072		6,168	,000
	N	,000	,001	,027	,344	,731
	w	,003	,001	,331	3,616	,000
	kmax	,001	,001	,059	,748	,456
	pe	,737	,119	,565	6,208	,000

a Dependent Variable: relent

Table A4 Results of the regression analysis for innovation

Model Summary

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate
1	,248(a)	,062	,061	370,06907
2	,261(b)	,068	,066	368,95179
3	,332(c)	,110	,107	360,77226
4	,334(d)	,111	,108	360,67821
5	,379(e)	,144	,139	354,23942
6	,379(f)	,144	,139	354,34778
7	,422(g)	,178	,172	347,40028

a Predictors: (Constant), N
b Predictors: (Constant), N, w
c Predictors: (Constant), N, w, kmax
d Predictors: (Constant), N, w, kmax, pe
e Predictors: (Constant), N, w, kmax, pe, p0
f Predictors: (Constant), N, w, kmax, pe, p0, delta
g Predictors: (Constant), N, w, kmax, pe, p0, delta, rho

Coefficients(a)

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.
		B	Std. Error	Beta		
1	(Constant)	-46,742	24,310		-1,923	,055
	N	3,303	,408	,248	8,092	,000
2	(Constant)	10,874	32,527		,334	,738
	N	3,251	,407	,244	7,979	,000
	w	-1,077	,406	-,081	-2,656	,008
3	(Constant)	142,344	37,170		3,830	,000
	N	3,263	,398	,245	8,190	,000
	w	-1,009	,397	-,076	-2,545	,011
	kmax	-2,665	,390	-,204	-6,835	,000
4	(Constant)	116,782	42,555		2,744	,006
	N	3,273	,398	,246	8,216	,000
	w	-1,003	,397	-,076	-2,528	,012
	kmax	-2,658	,390	-,204	-6,817	,000
	pe	47,530	38,558	,037	1,233	,218
5	(Constant)	4,741	45,624		,104	,917
	N	3,338	,391	,251	8,529	,000
	w	-1,100	,390	-,083	-2,822	,005
	kmax	-2,698	,383	-,207	-7,046	,000
	pe	39,817	37,891	,031	1,051	,294
	p0	237,817	38,836	,180	6,124	,000
6	(Constant)	16,527	49,367		,335	,738
	N	3,337	,392	,251	8,523	,000
	w	-1,097	,390	-,083	-2,813	,005
	kmax	-2,701	,383	-,207	-7,050	,000
	pe	39,901	37,903	,031	1,053	,293
	p0	238,198	38,852	,180	6,131	,000
	delta	-,242	,386	-,018	-,626	,531
7	(Constant)	-116,388	52,651		-2,211	,027
	N	3,348	,384	,252	8,722	,000
	w	-1,004	,383	-,076	-2,624	,009
	kmax	-2,670	,376	-,205	-7,108	,000
	pe	38,683	37,160	,030	1,041	,298
	p0	229,381	38,115	,174	6,018	,000
	delta	-,105	,379	-,008	-,276	,783
	rho	12,276	1,915	,185	6,412	,000

a Dependent Variable: ino

Table A5 Results of the regression analysis for innovation with empty networks excluded

Model Summary

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate
1	,336(a)	,113	,110	575,44381
2	,336(b)	,113	,107	576,23285
3	,453(c)	,205	,198	546,16160
4	,472(d)	,222	,213	541,10771
5	,528(e)	,278	,267	522,05566
6	,528(f)	,279	,266	522,73962
7	,576(g)	,332	,318	503,79783

a Predictors: (Constant), N
b Predictors: (Constant), N, w
c Predictors: (Constant), N, w, kmax
d Predictors: (Constant), N, w, kmax, pe
e Predictors: (Constant), N, w, kmax, pe, p0
f Predictors: (Constant), N, w, kmax, pe, p0, dela
g Predictors: (Constant), N, w, kmax, pe, p0, dela, rho

Coefficients(a)

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.
		B	Std. Error	Beta		
1	(Constant)	-127,528	76,244		-1,673	,095
	N	7,765	1,191	,336	6,519	,000
2	(Constant)	-137,818	84,273		-1,635	,103
	N	7,736	1,197	,334	6,463	,000
	w	,314	1,088	,015	,288	,773
3	(Constant)	222,524	98,627		2,256	,025
	N	8,080	1,136	,349	7,113	,000
	w	-1,550	1,074	-,074	-1,443	,150
	kmax	-6,989	1,122	-,317	-6,228	,000
4	(Constant)	27,220	121,695		,224	,823
	N	8,049	1,125	,348	7,151	,000
	w	-,761	1,104	-,036	-,689	,491
	kmax	-5,837	1,191	-,265	-4,900	,000
	pe	279,492	103,805	,142	2,692	,007
5	(Constant)	-219,712	127,123		-1,728	,085
	N	8,194	1,086	,354	7,544	,000
	w	-,854	1,065	-,041	-,802	,423
	kmax	-5,984	1,150	-,271	-5,205	,000
	pe	230,781	100,610	,117	2,294	,022
	p0	516,369	101,908	,238	5,067	,000
6	(Constant)	-211,582	129,206		-1,638	,102
	N	8,156	1,093	,352	7,466	,000
	w	-,724	1,124	-,034	-,645	,520
	kmax	-5,784	1,274	-,262	-4,539	,000
	pe	243,319	106,389	,123	2,287	,023
	p0	513,486	102,345	,236	5,017	,000
	dela	-,437	1,193	-,019	-,367	,714
7	(Constant)	-504,517	137,009		-3,682	,000
	N	8,005	1,053	,346	7,600	,000
	w	-,558	1,084	-,027	-,515	,607
	kmax	-5,269	1,232	-,239	-4,276	,000
	pe	258,070	102,574	,131	2,516	,012
	p0	471,535	98,975	,217	4,764	,000
	dela	-,304	1,150	-,013	-,264	,792
	rho	25,129	4,902	,233	5,127	,000

a Dependent Variable: ino

Table A6 Results of the regression analysis for average degree with empty networks excluded

Model Summary

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate
1	,262(a)	,068	,066	18,11549
2	,262(b)	,069	,063	18,13944
3	,441(c)	,195	,188	16,89233
4	,443(d)	,196	,187	16,90341
5	,443(e)	,196	,184	16,92799
6	,447(f)	,200	,186	16,91190
7	,548(g)	,300	,285	15,84625

a Predictors: (Constant), N
b Predictors: (Constant), N, w
c Predictors: (Constant), N, w, kmax
d Predictors: (Constant), N, w, kmax, pe
e Predictors: (Constant), N, w, kmax, pe, p0
f Predictors: (Constant), N, w, kmax, pe, p0, dela
g Predictors: (Constant), N, w, kmax, pe, p0, dela, rho

Coefficients(a)

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.
		B	Std. Error	Beta		
1	(Constant)	2,254	2,400		,939	,348
	N	,186	,037	,262	4,962	,000
2	(Constant)	1,872	2,653		,706	,481
	N	,185	,038	,260	4,909	,000
	w	,012	,034	,018	,340	,734
3	(Constant)	14,793	3,050		4,849	,000
	N	,197	,035	,277	5,616	,000
	w	-,055	,033	-,085	-1,661	,098
	kmax	-,251	,035	-,370	-7,221	,000
4	(Constant)	13,092	3,802		3,444	,001
	N	,197	,035	,277	5,605	,000
	w	-,048	,034	-,075	-1,401	,162
	kmax	-,241	,037	-,355	-6,465	,000
	pe	2,434	3,243	,040	,751	,453
5	(Constant)	13,395	4,122		3,249	,001
	N	,197	,035	,277	5,589	,000
	w	-,048	,035	-,075	-1,396	,164
	kmax	-,240	,037	-,355	-6,449	,000
	pe	2,494	3,262	,041	,764	,445
	p0	-,633	3,304	-,009	-,191	,848
6	(Constant)	14,311	4,180		3,423	,001
	N	,193	,035	,271	5,449	,000
	w	-,034	,036	-,052	-,923	,357
	kmax	-,218	,041	-,321	-5,284	,000
	pe	3,906	3,442	,064	1,135	,257
	p0	-,958	3,311	-,014	-,289	,773
	dela	-,049	,039	-,071	-1,277	,203
7	(Constant)	2,005	4,309		,465	,642
	N	,186	,033	,262	5,622	,000
	w	-,027	,034	-,041	-,780	,436
	kmax	-,196	,039	-,290	-5,062	,000
	pe	4,526	3,226	,075	1,403	,162
	p0	-2,720	3,113	-,041	-,874	,383
	dela	-,044	,036	-,063	-1,207	,228
	rho	1,056	,154	,319	6,847	,000

a Dependent Variable: deg

Table A7 Results of the regression analysis for the clustering coefficient with empty networks excluded

Model Summary

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate
1	,048(a)	,002	-,001	,15580
2	,099(b)	,010	,004	,15544
3	,439(c)	,193	,186	,14053
4	,458(d)	,210	,200	,13927
5	,458(e)	,210	,198	,13945
6	,470(f)	,221	,207	,13871
7	,616(g)	,379	,366	,12404

a Predictors: (Constant), N
b Predictors: (Constant), N, w
c Predictors: (Constant), N, w, kmax
d Predictors: (Constant), N, w, kmax, pe
e Predictors: (Constant), N, w, kmax, pe, p0
f Predictors: (Constant), N, w, kmax, pe, p0, dela
g Predictors: (Constant), N, w, kmax, pe, p0, dela, rho

Coefficients(a)

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.
		B	Std. Error	Beta		
1	(Constant)	,132	,021		6,384	,000
	N	,000	,000	,048	,885	,377
2	(Constant)	,116	,023		5,123	,000
	N	,000	,000	,041	,749	,455
	w	,000	,000	,087	1,593	,112
3	(Constant)	,246	,025		9,690	,000
	N	,000	,000	,062	1,250	,212
	w	,000	,000	-,038	-,731	,465
	kmax	-,003	,000	-,446	-8,696	,000
4	(Constant)	,296	,031		9,435	,000
	N	,000	,000	,063	1,289	,198
	w	,000	,000	-,075	-1,417	,157
	kmax	-,003	,000	-,498	-9,143	,000
	pe	-,071	,027	-,141	-2,657	,008
5	(Constant)	,300	,034		8,844	,000
	N	,000	,000	,063	1,277	,202
	w	,000	,000	-,075	-1,409	,160
	kmax	-,003	,000	-,497	-9,119	,000
	pe	-,070	,027	-,139	-2,606	,010
	p0	-,010	,027	-,018	-,369	,712
6	(Constant)	,313	,034		9,126	,000
	N	,000	,000	,053	1,077	,282
	w	,000	,000	-,037	-,670	,503
	kmax	-,002	,000	-,442	-7,366	,000
	pe	-,051	,028	-,100	-1,794	,074
	p0	-,015	,027	-,026	-,534	,594
	dela	-,001	,000	-,118	-2,136	,033
7	(Constant)	,184	,034		5,461	,000
	N	,000	,000	,042	,948	,344
	w	,000	,000	-,024	-,476	,635
	kmax	-,002	,000	-,402	-7,465	,000
	pe	-,044	,025	-,088	-1,749	,081
	p0	-,033	,024	-,059	-1,351	,177
	dela	-,001	,000	-,108	-2,181	,030
	rho	,011	,001	,401	9,147	,000

a Dependent Variable: cls

Table A8 Results of the regression analysis for the evolution of the networks

Model Summary

Step	-2 Log likelihood	Cox & Snell R Square	Nagelkerke R Square
1	804,104(a)	,411	,549

a Estimation terminated at iteration number 5 because parameter estimates changed by less than ,001.

Variables in the Equation

		B	S.E.	Wald	df	Sig.	Exp(B)
Step 1(a)	N	,011	,003	12,121	1	,000	1,011
	w	-,041	,004	133,447	1	,000	,960
	kmax	-,042	,004	139,799	1	,000	,959
	pe	-2,910	,333	76,536	1	,000	,054
	p0	,096	,307	,099	1	,753	1,101
	delta	,011	,003	13,063	1	,000	1,011
	rho	,033	,004	87,659	1	,000	1,034
	Constant	3,305	,436	57,435	1	,000	27,242

a Variable(s) entered on step 1: N, w, kmax, pe, p0, delta, rho.

Table A9 Results of the regression analysis for the average change in relative entropy

Model Summary

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate
1	,013(a)	,000	-,002	,00172
2	,132(b)	,018	,014	,00171
3	,136(c)	,019	,013	,00171
4	,277(d)	,077	,070	,00166
5	,285(e)	,081	,073	,00166
6	,285(f)	,081	,071	,00166
7	,285(g)	,081	,069	,00166

a Predictors: (Constant), N
b Predictors: (Constant), N, w
c Predictors: (Constant), N, w, kmax
d Predictors: (Constant), N, w, kmax, pe
e Predictors: (Constant), N, w, kmax, pe, p0
f Predictors: (Constant), N, w, kmax, pe, p0, delta
g Predictors: (Constant), N, w, kmax, pe, p0, delta, rho

Coefficients(a)

		Unstandardized Coefficients		Standardized Coefficients		
Model		B	Std. Error	Beta	t	Sig.
1	(Constant)	,000	,000		2,213	,027
	N	8,09E-007	,000	,013	,303	,762
2	(Constant)	,001	,000		3,510	,000
	N	1,17E-006	,000	,019	,443	,658
	w	-7,92E-006	,000	-,132	-3,085	,002
3	(Constant)	,001	,000		2,265	,024
	N	1,10E-006	,000	,018	,416	,678
	w	-7,28E-006	,000	-,121	-2,690	,007
	kmax	2,20E-006	,000	,034	,753	,452

4	(Constant)	,001	,000		5,132	,000
	N	9,97E-007	,000	,016	,387	,699
	w	-1,02E-005	,000	-,171	-3,829	,000
	kmax	-1,87E-006	,000	-,029	-,640	,522
	pe	-,001	,000	-,250	-5,805	,000
5	(Constant)	,002	,000		5,344	,000
	N	1,01E-006	,000	,016	,392	,695
	w	-1,06E-005	,000	-,177	-3,957	,000
	kmax	-1,84E-006	,000	-,028	-,629	,530
	pe	-,001	,000	-,250	-5,808	,000
	p0	,000	,000	-,068	-1,635	,103
6	(Constant)	,002	,000		5,204	,000
	N	9,99E-007	,000	,016	,387	,699
	w	-1,05E-005	,000	-,176	-3,883	,000
	kmax	-1,71E-006	,000	-,026	-,569	,570
	pe	-,001	,000	-,249	-5,739	,000
	p0	,000	,000	-,068	-1,640	,102
	delta	-5,13E-007	,000	-,009	-,200	,842
7	(Constant)	,002	,000		4,951	,000
	N	9,98E-007	,000	,016	,387	,699
	w	-1,05E-005	,000	-,176	-3,861	,000
	kmax	-1,71E-006	,000	-,026	-,568	,570
	pe	-,001	,000	-,249	-5,707	,000
	p0	,000	,000	-,068	-1,638	,102
	delta	-5,13E-007	,000	-,009	-,200	,842
	rho	2,95E-008	,000	,000	,011	,991

a Dependent Variable: relent_avgch

Table A10 Results of the regression analysis for the evolution of relative innovation

Model Summary

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate
1	,016(a)	,000	-,002	2,98140
2	,050(b)	,003	-,001	2,98084
3	,066(c)	,004	-,001	2,98076
4	,116(d)	,014	,006	2,96990
5	,117(e)	,014	,004	2,97248
6	,130(f)	,017	,006	2,97050
7	,143(g)	,020	,008	2,96777

a Predictors: (Constant), N
b Predictors: (Constant), N, w
c Predictors: (Constant), N, w, kmax
d Predictors: (Constant), N, w, kmax, pe
e Predictors: (Constant), N, w, kmax, pe, p0
f Predictors: (Constant), N, w, kmax, pe, p0, delta
g Predictors: (Constant), N, w, kmax, pe, p0, delta, rho

Coefficients(a)

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.
		B	Std. Error	Beta		
1	(Constant)	,666	,287		2,316	,021
	N	-,002	,005	-,016	-,381	,703

2	(Constant)	,475	,336		1,416	,157
	N	-,002	,005	-,019	-,434	,665
	w	,005	,004	,047	1,097	,273
3	(Constant)	,749	,430		1,739	,083
	N	-,002	,005	-,017	-,400	,689
	w	,003	,005	,033	,724	,470
	kmax	-,005	,005	-,046	-1,015	,311
4	(Constant)	,129	,512		,252	,801
	N	-,002	,005	-,017	-,397	,692
	w	,005	,005	,053	1,142	,254
	kmax	-,002	,005	-,021	-,460	,646
	pe	,988	,445	,099	2,220	,027
5	(Constant)	,195	,568		,343	,732
	N	-,002	,005	-,017	-,397	,692
	w	,005	,005	,052	1,116	,265
	kmax	-,002	,005	-,021	-,458	,647
	pe	,989	,446	,099	2,220	,027
	p0	-,119	,446	-,012	-,268	,789
6	(Constant)	-,010	,588		-,016	,987
	N	-,002	,005	-,016	-,364	,716
	w	,004	,005	,043	,913	,362
	kmax	-,004	,005	-,035	-,736	,462
	pe	,918	,448	,092	2,047	,041
	p0	-,097	,446	-,009	-,217	,829
	delta	,006	,005	,058	1,309	,191
7	(Constant)	,256	,617		,414	,679
	N	-,002	,005	-,014	-,331	,741
	w	,005	,005	,050	1,056	,292
	kmax	-,004	,005	-,032	-,659	,510
	pe	,983	,450	,098	2,183	,029
	p0	-,081	,446	-,008	-,181	,857
	delta	,006	,005	,058	1,319	,188
	rho	-,007	,005	-,061	-1,408	,160

a Dependent Variable: relino

Table A11 Results of the regression analysis for the evolution of relative degree

Model Summary

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate
1	,031(a)	,001	-,001	1,13394
2	,034(b)	,001	-,003	1,13504
3	,039(c)	,002	-,005	1,13606
4	,040(d)	,002	-,007	1,13726
5	,040(e)	,002	-,009	1,13848
6	,061(f)	,004	-,009	1,13849
7	,068(g)	,005	-,010	1,13915

a Predictors: (Constant), N
b Predictors: (Constant), N, w
c Predictors: (Constant), N, w, kmax
d Predictors: (Constant), N, w, kmax, pe
e Predictors: (Constant), N, w, kmax, pe, p0
f Predictors: (Constant), N, w, kmax, pe, p0, delta
g Predictors: (Constant), N, w, kmax, pe, p0, delta, rho

Coefficients(a)

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.
		B	Std. Error	Beta		
1	(Constant)	,227	,114		1,984	,048
	N	,001	,002	,031	,679	,497
2	(Constant)	,204	,136		1,502	,134
	N	,001	,002	,031	,670	,503
	w	,001	,002	,014	,311	,756
3	(Constant)	,246	,173		1,422	,156
	N	,001	,002	,032	,686	,493
	w	,000	,002	,009	,180	,857
	kmax	-,001	,002	-,019	-,394	,693
4	(Constant)	,230	,209		1,103	,271
	N	,001	,002	,032	,687	,493
	w	,000	,002	,010	,204	,839
	kmax	-,001	,002	-,017	-,350	,727
	pe	,025	,183	,007	,139	,890
5	(Constant)	,235	,230		1,018	,309
	N	,001	,002	,032	,685	,493
	w	,000	,002	,010	,201	,841
	kmax	-,001	,002	-,017	-,348	,728
	pe	,025	,183	,007	,139	,890
	p0	-,008	,185	-,002	-,045	,964
6	(Constant)	,298	,239		1,247	,213
	N	,001	,002	,031	,671	,502
	w	,001	,002	,018	,350	,727
	kmax	,000	,002	-,007	-,129	,897
	pe	,046	,184	,012	,249	,804
	p0	-,012	,185	-,003	-,066	,948
	delta	-,002	,002	-,047	-,995	,320
7	(Constant)	,246	,251		,981	,327
	N	,001	,002	,030	,651	,515
	w	,001	,002	,014	,282	,778
	kmax	,000	,002	-,009	-,167	,867
	pe	,031	,186	,008	,168	,866
	p0	-,017	,186	-,004	-,091	,928
	delta	-,002	,002	-,047	-,994	,321
	rho	,001	,002	,032	,680	,497

a Dependent Variable: reldeg

Table A12 Results of the regression analysis for the evolution of relative clustering

Model Summary

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate
1	,016(a)	,000	-,002	,39588
2	,039(b)	,002	-,003	,39610
3	,085(c)	,007	,000	,39544
4	,091(d)	,008	-,001	,39571
5	,091(e)	,008	-,004	,39618
6	,154(f)	,024	,010	,39356
7	,172(g)	,030	,013	,39283

a Predictors: (Constant), N
b Predictors: (Constant), N, w
c Predictors: (Constant), N, w, kmax
d Predictors: (Constant), N, w, kmax, pe
e Predictors: (Constant), N, w, kmax, pe, p0
f Predictors: (Constant), N, w, kmax, pe, p0, delta
g Predictors: (Constant), N, w, kmax, pe, p0, delta, rho

Coefficients(a)

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.
		B	Std. Error	Beta		
1	(Constant)	,240	,042		5,660	,000
	N	,000	,001	,016	,341	,734
2	(Constant)	,260	,051		5,137	,000
	N	,000	,001	,016	,340	,734
	w	,000	,001	-,036	-,737	,462
3	(Constant)	,321	,064		5,015	,000
	N	,000	,001	,020	,420	,674
	w	-,001	,001	-,060	-1,183	,237
	kmax	-,001	,001	-,079	-1,551	,122
4	(Constant)	,293	,077		3,801	,000
	N	,000	,001	,021	,432	,666
	w	-,001	,001	-,053	-1,026	,306
	kmax	-,001	,001	-,071	-1,353	,177
	pe	,044	,067	,033	,661	,509
5	(Constant)	,291	,085		3,438	,001
	N	,000	,001	,021	,431	,667
	w	-,001	,001	-,053	-1,022	,307
	kmax	-,001	,001	-,071	-1,352	,177
	pe	,044	,067	,033	,661	,509
	p0	,002	,067	,002	,034	,973
6	(Constant)	,351	,087		4,020	,000
	N	,000	,001	,019	,392	,695
	w	,000	,001	-,033	-,625	,532
	kmax	-,001	,001	-,042	-,796	,427
	pe	,066	,067	,050	,989	,323
	p0	-,002	,067	-,002	-,032	,975
	delta	-,002	,001	-,128	-2,574	,010
7	(Constant)	,308	,091		3,382	,001
	N	,000	,001	,017	,348	,728
	w	-,001	,001	-,041	-,785	,433
	kmax	-,001	,001	-,047	-,879	,380
	pe	,052	,067	,039	,771	,441
	p0	-,007	,067	-,005	-,099	,921
	delta	-,002	,001	-,128	-2,585	,010
	rho	,001	,001	,078	1,604	,110

a Dependent Variable: relcls